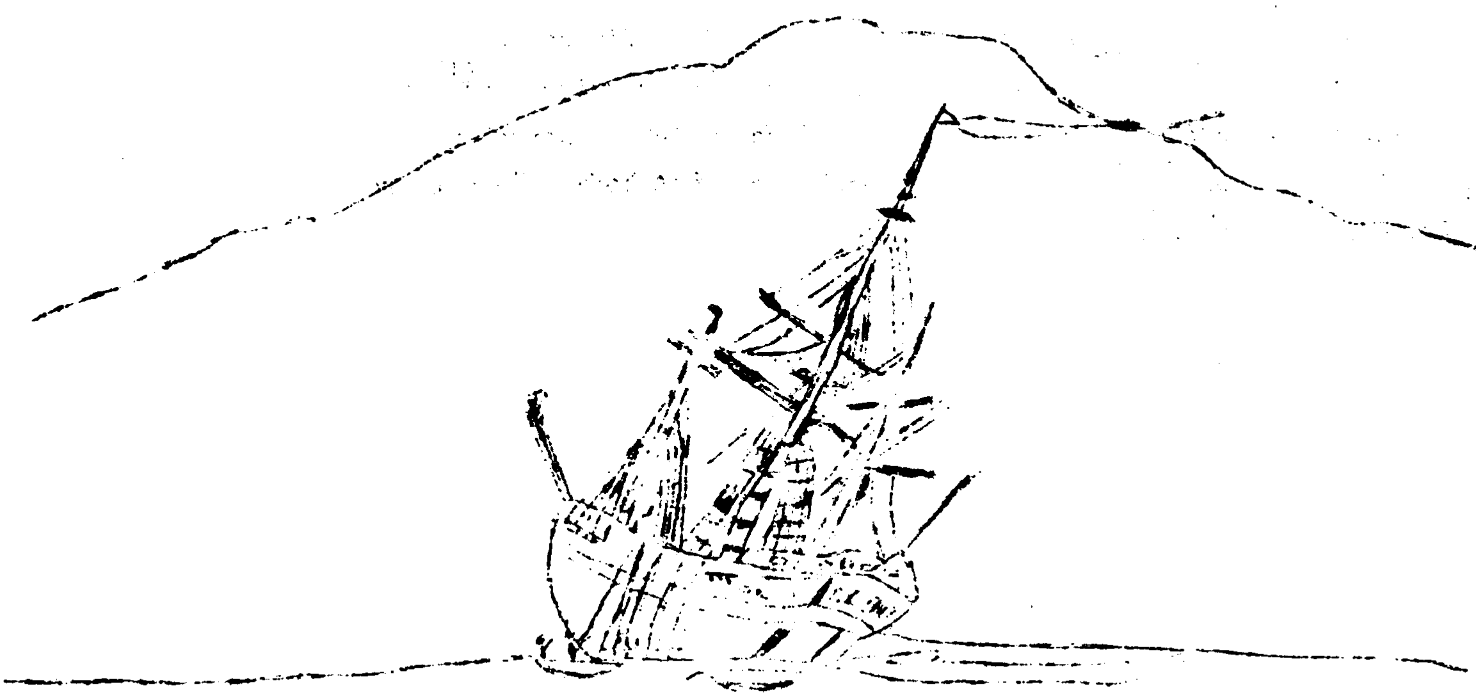
# JAMES COOK MATHEMATICAL NOTES

## ISSUE NUMBER 10 IN JULY 1977.



This picture (after a drawing by Sydney Parkinson) shows HMS Endeavour careened for repairs in the mouth of the Endeavour River where Cooktown is now built. This was in July 1770.

## TIDES

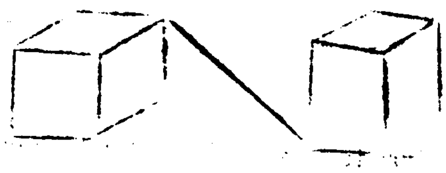
The following ideas come from H.O. Davies of the J.C.U.N.Q.

On 11th June 1770 at 11pm. HMS Endeavour ran aground on a coral reef. The tide was high and falling. The crew worked through the night lightening the ship in the hope that she would float off on the morning tide, but the water did not rise high enough. During the afternoon however, Cook had kedge anchors laid out and on the evening tide she was pulled off. Did Captain Cook know that the evening tide would rise higher than the morning tide? He had with him Charles Green the astronomer, and they had been taking lunar distances in order to find longitude, so that they would certainly know the position of the moon. Newton's inverse square law of gravitation would be familiar certainly to Green and possibly also to Cook. From it there is no serious difficulty in finding the movement of the gravitational equipotential surface above and below mean sea level (this is Newton's equilibrium theory of the tides). Had Cook and Green ever talked about tidal theory in the 22 months they had been together? Cook described his tidal observations at Endeavour River in the Philosophical Transactions of the Royal Society, Vol. 66, p 447.

## THE WINDOW FRAME (JCMN 8)

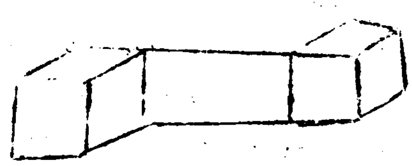
From C.F. Moppert. Euler's formula in n dimensions is $d^{(0)} - d^{(1)} + \dots \pm d^{(n)} = 1$ where $d^{(0)}$ is the number of vertices, $d^{(1)}$ the number of edges, ... and $d^{(n)}$ the number of *"solids"* which is normally one. The result is well-known for convex polytopes but to what bodies (apart from window frames) will it extend?

*In three dimensions if we connect two cubes by an edge*



$v-e+f-s = 16-25+12-2 = 1$

*and if we connect them by an additional quadrilateral face*



$v-e+f-s = 16-26+13-2 = 1$

*My question is then: when is* $v-e+f-s = 1$? *or in two dimensions when is* $v-e+f = 1$?

## A LITTLE PROBLEM FROM LONDON (JCMN 9)

Given a square matrix $A$, there is the set $E$ of all complex $k$ for which the matrices $A$ and $kA$ are similar. What is it?

No complete answers have come in. It appears that there are four possibilities, the set $E$ of all $k$ may be (a) *the whole complex plane, or* (b) *the whole complex plane except zero, or* (c) *for any positive integer* $n$ *the set of* $n^{th}$ *roots of unity, or* (d) *the number one only.*
Are there any other possibilities?

## INEQUALITY WANTED

The following problem by Kenneth S. Williams of Carleton University, Ottawa is reprinted from Eureka where it appeared as number 247 on page 131 (May 1977).

On page 215 of Analytic Inequalities by D.S. Mitrinović the following inequality is given: if $0 < b \leq a$ then

$$\frac{(a-b)^2}{8a} \leq \frac{a+b}{2} - \sqrt{ab} \leq \frac{(a-b)^2}{8b}$$

Can this be generalised to the following form: if $0 < a_1 \leq \ldots \leq a_n$ then

$$(k/a_n) \sum_{1 \leq i < j \leq n} (a_i - a_j)^2 \leq \frac{a_1 + \ldots + a_n}{n} - (a_1 \ldots a_n)^{1/n} \leq (k/a_1) \sum_{1 \leq i < j \leq n} (a_i - a_j)^2$$

where $k$ is a function of $n$ ?

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## CONGRUENT QUADRILATERALS (JCMN 8)

Given an ellipse, find two sets of four points, ABCD and A'B'C'D' on it such that ABCD is congruent to A'B'C'D' but not obtained from it by one of the trivial symmetries of the ellipse.

H.O. Davies solves this problem by taking an ellipse congruent to the first and meeting it in four points and having a different centre from the first. Pick up the second ellipse with the four points marked on it, and lay it down to coincide with the first ellipse.

The proposer, G. Szekeres, adds the comment that instead of an ellipse we can have any closed convex curve except a circle, the same method will apply.

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## HIGH POWER MATRIX METHODS FOR FIRST YEARS

Find $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 2i & 0 \\ 0 & 0 & i \end{pmatrix}^{100} \begin{pmatrix} i \\ 1 \\ 0 \end{pmatrix}$

R.L. Agacy

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## A QUOTATION

Give me a fruitful error any time, full of seeds, bursting with its own corrections. You can keep your sterile truth for yourself.

Vilfredo Pereto (1848-1923)



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IS TRIGONOMETRY STILL TAUGHT? (JCMN 9)

Evidently not, for your editor has not had any solution sent in. Question 9 was as follows:

> Having given that $\sin x = n \sin(x+\alpha)$, prove that when $n< 1$,
> $x + k\pi = \sin\alpha + \frac{1}{2} n^2 \sin 2\alpha + \ldots$
>
> where k is some integer or zero.

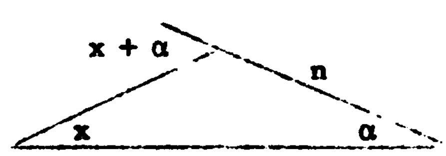
Explain how the value of k is to be chosen. Also find a series for y in terms of ascending powers of $\cos\alpha$, where

$$2 \tan y = \sin x \operatorname{cosec} \frac{x + \alpha}{2} \operatorname{cosec} \frac{x - \alpha}{2} .$$

(Trinity College Cambridge, Examination in Trigonometry, 9-12, Wednesday, June 7th, 1893.)

The proper way to approach this question is to try to get insight into the nature of a typical candidate for the examination. This young gentleman of Trinity College may have been in the College Chapel on the Advent Sunday when Dr. Montagu Butler preached on the parable of the ten virgins (Chapter 25 of St. Matthew's Gospel) and ended with the rhetorical question "Would you, my dear young friends, like to be inside with the five wise virgins, or outside, alone and in the dark, with the five foolish ones?" Episodes like this, while brightening his Sunday would have done nothing for his mathematics, but if he had ever been to St. Andrew's Church at the end of Petty Cury he would no doubt have noticed the plaque recording the burial there of Captain Cook's wife and his sons James and Hugh. This would remind our hypothetical Trinity undergraduate of how the great age of exploration was made possibly by astronomical navigation which in turn depended on spherical trigonometry.

He would probably have learnt Greek at school and so would know that trigonometry was done with triangles. The given relation between n, x and $\alpha$ is an application of the sine rule to this triangle:



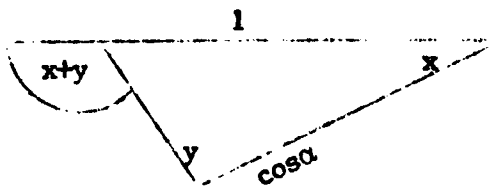

A good piece of general knowledge is the formula

$$\log(1-z) = -(z + \tfrac{1}{2}z^2 + z^3/3 + \ldots)$$

valid if $|z| < 1$ for the single-valued logarithm function that has imaginary part between plus and minus $\pi/2$ in the right half plane. Putting $z = n \exp(-i\alpha)$

and taking the imaginary part in this formula gives the required result, with k the nearest integer to $-x/\pi$.

The second part of the question, as is usual in examinations, presents no serious difficulty to those who have done the first part. The relation connecting, x, y and α is illustrated by the triangle:




Then $x + y$ (plus some multiple of π) is the imaginary part of $\log(1-e^{ix}\cos\alpha)$ and therefore (for some integer m)

$$y = m\pi - x - \cos\alpha \sin x = \tfrac{1}{2} \cos^2\alpha \sin 2x - \ldots$$

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<u>STOP PRESS</u>. Trigonometry has been taught, for J.B. Parker has sent in a solution like the one above. He expressed doubt whether the second series was what was wanted, should it be transformed to involve only the n (as given by the first part) instead of x in the coefficients of the powers of cosα? Your editor thinks not, for reasons as follows. Suppose if possible there were an expansion of $y = y_2(n, \cos\alpha)$ equivalent to the one $y = y_1(x, \cos\alpha)$ obtained above. Write the relation $\sin x = n \sin(x + \alpha)$, as $n = n(x, \alpha)$, then clearly $n(-x, -\alpha) = n(x, \alpha)$. It follows that $y_1(x, \cos\alpha) = y = y_2\big(n(x, \alpha), \cos\alpha\big) = y_2\big(n(-x, -\alpha), \cos(-\alpha)\big) = y_1(-x, \cos\alpha) = -y_1(x, \cos\alpha)$, which is a contradiction because y is not identically zero.

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COMBINATORIAL GEOMETRY PROBLEM

Given $\tfrac{1}{2}n(n - 1)$ unequal positive real numbers $r(i, j)$ for $1 \le i < j \le n$, can you find n points in the plane such that the $\tfrac{1}{2}n(n - 1)$ distances $d(i, j)$ between pairs of them are ordered in the same way as the $r(i, j)$? That is $d(i, j) < d(p, q)$ if and only if $r(i, j) < r(p, q)$.

A.J. DOBSON.

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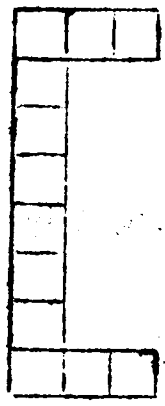
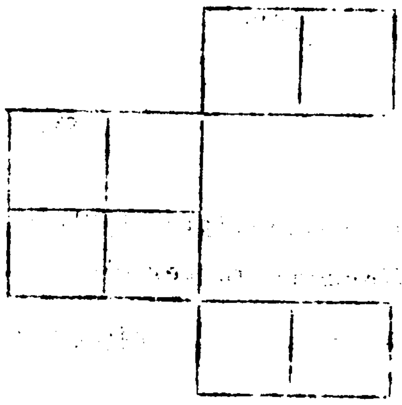
## CENTROIDS (JCMN 6 and 9)

A question asked in "The centre of Australia" (JCMN 6) was "For what n is there a plane figure that can be bisected in just n ways by straight lines through the centroid?" No solutions have yet come in. The following examples, each made up of unit squares, deal with the cases n = 1 and n = 2.





For n = 3, 5, 7, ... there is the example of the regular n-gon, which is bisected by each of its n lines of symmetry.

H.O. Davies comments that for compact convex sets the property of being bisected by all lines through the centroid is a necessary and sufficient condition for central symmetry (that is invariance under a rotation of half a revolution about the centroid).

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## AN ENQUIRY

Does any reader know of a book that obtains Lagrange's equations for electric circuits and for electro-mechanical systems? Preferably written for second year students and treating only the quasi-static case where the speed of light is approximated by infinity?

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## MORE TRIGONOMETRY

Find a family of equations for sums of squares of tangents starting with:

$$\tan^2 60^\circ = 3$$

$$\tan^2 36^\circ + \tan^2 72^\circ = 10$$

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## CAPTAIN COOK'S MOUNTAINS (JCMN 9)

We are indebted to H.T. Fry of the History Department of The James Cook University for the following extracts from the Journal of Captain Cook.

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Sunday 22nd April 1770.

At Noon we were by observation in the Latitude of 35° 27' and Long$^{de}$ 209° 23'.... A remarkable peaked hill laying inland the top of which look'd like a <u>Pigeon House</u> and occasioned my giving it that name, bore N 32° 30' W.....

Wednesday 16th May 1770. Winds southerly a fresh gale with which we steerd North along shore untill sun-set at which time we discoverd breakers ahead and on our larboard bow, being at this time in 20 fathom water and about 5 Miles from the land. Hauld off east untill 8 oClock at which time we had run 8 Miles and had increased our depth of water to 44 fathoms. We then brought too with her head to the Eastward and lay on this tack untill 10 oClock when having increased our soundings to 78 fathoms we wore and lay with her head in shore untill 5 oClock AM when we made sail. At day light we were Surprised in finding our selves farther to the southward than we were in the evening and yet it had blowen strong Southerly all night. We now saw the breakers again within us which we past at the distance of 1 League, they lay in the Lat$^{de}$ of 38° 8' (Here Capt Cook made a mistake, it should be 28, not 38) and stretch off East two Leagues from a point under which is a small Island, there situation may always be found by the peaked mountain before mentioned which bears S W B W from them and on this account I have named Mount Warning. It lies 7 or 8 Leagues inland in the latitude of 28° 22' S, the land is high and hilly about it but it is conspicuous enough to be distinguished from every thing else.

*****

The gas helium was so named by J.N. Lockyer in 1868, having inferred from observations of the chromosphere that a gas with certain spectral lines existed on the surface of the sun. The first evidence of helium on earth was found by a scientist of the U.S. geological survey in 1891 when examining a gas obtained by heating uranite, but unfortunately he regarded the observation as experimental error. The effective discovery of helium therefore had to wait for Sir William Ramsay's investigations in 1895.

Captain Cook was like a good scientist or a good detective, when he found the ship too far south in the morning he carefully noted the fact, and he did not dismiss it as just evidence of inaccurate estimation of courses and speeds during the night. Now we know that he was right, the officers of the watch during that night had done their dead reckoning accurately, for there is a southerly set along the Eastern Australian coast that would have carried the ship southwards.

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## BOOK REVIEW

The Universal Encyclopedia of Mathematics, Pan Books, 1976, 715 pages, paperback, ISBN 0 330 24396 9, recommended price in Australia $4.30 and in U.K. £1.50.

This is a translation and revision of Meyers Rechenduden published in Mannheim in 1960. It consists mainly of an alphabetical set of paragraphs each giving a definition and other information about mathematical words and phrases, from "absolute value" to "zero". At the end are formulae and tables.

Minor criticisms are that the picture of the cissoid should be turned through one right angle, or the equations changed; the description of a loxodrome should mention that it is more usually called a rhumb line; and the paragraph on periodic decimal numbers should mention that they are rational. The entry for "Relative numbers" could well be omitted.

However the book has merit as a work of reference for students in first year at university or final year at school.

B.C.R.

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## APPLIED MATHEMATICS

Congratulations to A.A. Richardson of the Mathematics Department of JCUNQ. Following in the wake of Captain Cook he won the Townsville Cruising Yacht Club race to Cairns in his boat "Virgin Gold", on June 11th and 12th.

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*Your editor would like to hear from you anything connected with mathematics or with James Cook.*

*Prof. B.C. Rennie, Mathematics Department,*
*James Cook University of North Queensland,*
*Post Office James Cook University, Q.4811,*
*Australia.*