

Contents

1 Connections on principal fibre bundles	3
1.1 Motivation: the Dirac monopole	3
1.2 Principal fibre bundles	4
1.3 Connections	6
1.3.1 Connections as horizontal distributions	6
1.3.2 The connection one-form	8
1.3.3 Gauge fields	8
1.4 The space of connections	10
1.5 Gauge transformations	11
1.6 The action of \mathcal{G} on \mathcal{A}	11
2 Curvature	13
2.1 The curvature of a connection	13
2.1.1 The horizontal projection	13
2.1.2 The curvature 2-form	13
2.1.3 Gauge field-strengths	15
2.2 The covariant derivative	15
2.2.1 Basic forms	15
2.2.2 The covariant derivative	16
3 The Yang–Mills equations	19
3.1 Some geometry	19
3.1.1 The volume form	19
3.1.2 The Hodge \star operator	19
3.1.3 Inner product on bundle-valued forms	20
3.2 The variational problem	21
3.2.1 The action functional	21
3.2.2 The field equations	22
3.3 Coupling to matter	23
4 Instantons	24
4.1 (Anti-)self-duality	24
4.2 What is c ?	25
4.3 The Chern–Simons form	26
4.4 The BPST instanton	26
5 Instanton moduli space	28
5.1 Irreducible connections	28
5.2 The deformation complex	29