JAMES COOK MATHEMATICAL NOTES

Issue Number 3, prepared in March 1976 to celebrate the lecture given on 7th March 1776 by Captain James Cook R.N. to the Royal Society on the subject of scurvy. This lecture (Phil. trans. R.S. 66) and Captain Cook's work on the prevention of scurvy at sea were recognised by the award of the Copley Medal.

A portrait plaque of Cook by Josiah Wedgwood.

GEOMETRY WITHOUT COMPASSES

One of the skills occasionally needed by a teacher of mathematics is to make diagrams on the wax stencil sheets used in duplicating homework papers. To draw a good circle you need a coin of the right size. This suggests the pure mathematical problem of finding what geometrical constructions are possible without the traditional pair of compasses. For example using a ruler with parallel sides, given a line and the centre and radius of a circle, can you find the intersections of the circle with the line?

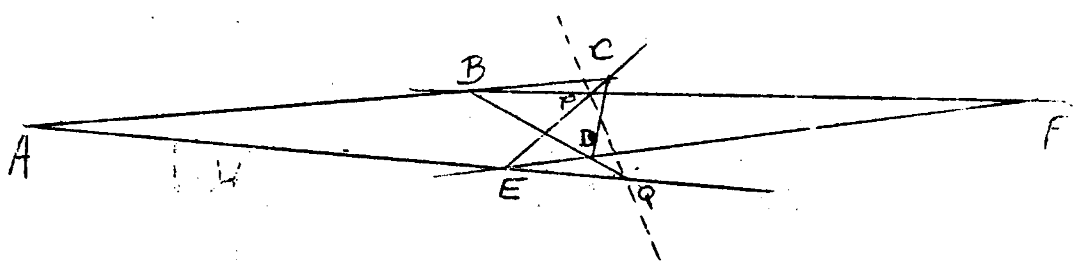
ANOTHER MATRIX PROBLEM

If B is a square matrix and I the unit matrix show that it is impossible to find a positive integer m and a non-zero vector $\underline{v}$ such that $B^m\underline{v} = (B + I)^m\underline{v} = 0$.

GEOMETRY MADE DIFFICULT

C.F. Moppert's problem was to join two points when your ruler is not long enough. This solution is from B.B. Newman. First note that a line may be produced to any length by moving the ruler along it.

To join A and F draw two lines through each, AB, AE, FB, FE, and take points C on AB and D on FE. With luck these new points will be close enough together to use your ruler. If not try again. Let P be where CE meets BF and Q where BD meets AE. Then PQ is the Pappus line for the two triples ABC and DEF, so that where PQ meets CD is on the line AF. In this way points on AF may be constructed until two of them are near enough to be joined with a ruler.



MORE HISTORY

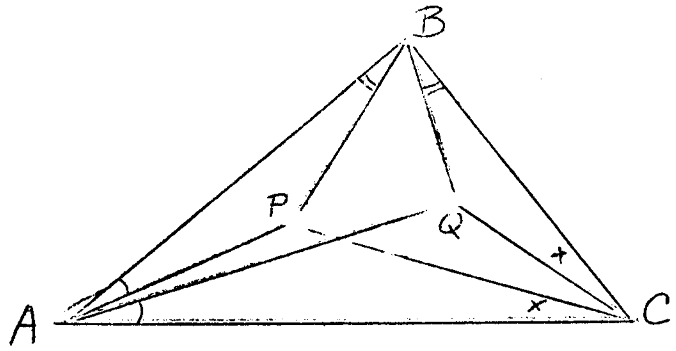
Magnetic Island, five miles North of Townsville, was discovered and named by Captain Cook in June, 1770, under the mistaken impression that the magnetic deviation changed suddenly near the island. I used to know Professor W.A. Osborne who lived there in his retirement after being Professor of Physiology at Melbourne University from 1904 to 1938. He used to say that he knew of nowhere with a healthier climate, he went to Melbourne and died in 1967. Professor Osborne once told me about Captain R.F. Scott R.N. visiting Melbourne in 1911 with H.M.S. Terra Nova, going to explore the Antarctic. The ship did not take on any fresh fruit or vegetables and Professor Osborne had asked Captain Scott about it. There were two reasons, one was that a Frenchman, L. Pasteur, had recently shown how diseases were caused by microbes, so that the old beliefs about scurvy grass and lime juice were rather discredited; the other reason was that the Admiralty had notified victualling [illegible] that some Melbourne merchants were untrustworthy. Professor Osborne thought that the sledge party who failed to get back from the Pole (Scott, Bowers, Evans, Oates and Wilson) were suffering from scurvy. In the fifty years before the Antarctic Expedition the Navy had changed from sail to steam, a network of coaling stations in every ocean had been set up, and long periods at sea had become unknown. Consequently the work of Cook on scurvy had been forgotten.

LATE NEWS

It is reported from England that Mr. Gordon Cook of Warwick and his wife, son and daughter plan to follow the track of Captain Cook's last voyage 200 years later, leaving Plymouth in the 70 foot schooner Wave-walker on the anniversary, 12th July, 1976.

POLYNOMIALS, ZEROS AND DERIVATIVES

In "Pure Geometry" by E.H. Askwith (C.U.P. 1903) there is explained on page 36 (second edition) the idea of isogonal conjugates with respect to a triangle ABC. Two lines through A are called isogonally conjugate if the bisectors of the angle between them also bisect BAC, in other words AF and AG are isogonally conjugate if the angles BAF and GAC (taking account of sign) are equal. There is a theorem (pages 36 - 37) that if P is any point then the isogonal conjugates of AP, BP and CP are concurrent.



Call the intersection Q the isogonal conjugate of P. The relation between P and Q is symmetric. The following are isogonal conjugates:

Orthocentre of ABC	Circumcentre of ABC
Any circle through A and B	Another circle through A and B
Circumcircle of ABC	Line at infinity
Centroid of ABC	Defined as symmedian point

Askwith gives various properties of the symmedian point in Exercises 9, 10, 74 and 15 (pages 39 and 40).

Now reverting to the questions in our first and second issue, let L and N be the zeros of the derivative of the cubic that has zeros at A, B and C. The comment of H.O. Davies is just that L and N are isogonal conjugates.

GETTING INTO CYCLES

Circulating in the mathematical underground for some years now has been the idea of sequences of integers $a_1, a_2, \ldots$ such that a_{n+1} is either $\frac{1}{2}a_n$ or $3a_n+1$ according to whether a_n is even or odd. It seems likely that all such sequences of positive integers are ultimately cyclic, having the three values 1, 4, 2 recurring. It is less well known that with negative integers there are three cycles, one of two elements, - 1, -2, one of five, -5, -14, -7, -20, -10 and one longer one starting with -17. Is there any other cycle? Is there a sequence not ultimately cyclic?

HAVE A GUESS

The following question arises from some work of J.M. Hammersley. The pair of equations

$$A(x) = \inf_y \,(B(y) + xy) \quad \text{and} \quad B(y) = \sup_x \,(A(x) - xy)$$

are satisfied for example by $A(x) = -\frac{1}{2}mx^2$ and $B(y) = \frac{1}{2}y^2/m$ for any positive m. In what circumstances do the equations give a reciprocal pair of transforms?

A HARDER MATRIX PROBLEM

Theorem (due to A.P. Guinand)

If
$$\begin{bmatrix} 0 & a_{12} & a_{13} \\ a_{21} & 0 & a_{23} \\ a_{31} & a_{32} & 0 \end{bmatrix} \begin{bmatrix} 0 & b_{12} & b_{13} \\ b_{21} & 0 & b_{23} \\ b_{31} & b_{32} & 0 \end{bmatrix} \begin{bmatrix} \lambda & c_{12} & c_{13} \\ c_{21} & \mu & c_{23} \\ c_{31} & c_{32} & \nu \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where none of the a's or b's is zero, then if two of λ, μ, ν are zero it follows that the third is zero.

Bonus marks for an elegant proof and for any generalization to higher order matrices.

THE AIRPORT WAITING ROOM

One day in 1972 while waiting to see off Laurent Schwartz and his wife Marie-Helene, I remembered a question that had been at the back of my mind for twenty years, I had never been able to solve it, and had even forgotten how the problem had arisen. When I put it to the great man he stared into the middle distance for about half a minute and then told me the answer. Why not borrow a stop-watch and try this on your friends?

Can a function in L^2 and its Fourier transform both vanish in an interval? (For the answer $f(x) = 0$ give one mark out of ten and suggest trying again.)

NOTICE

Professor R.B. Potts of Adelaide has agreed to be guest editor until B.C. Rennie comes back in October from study leave. Please send all correspondence and contributions to

Professor R.B. Potts,
Department of Applied Mathematics,
University of Adelaide,
ADELAIDE, S.A. 5001, Australia.