JAMES COOK MATHEMATICAL NOTES

Number 2, dated January 1976, celebrating the 200th anniversary of Captain Cook's holding an appointment at the Royal Naval Hospital at Greenwich (later re-named the Royal Naval College).

POLYNOMIALS, THEIR ZEROS AND DERIVATIVES

The following problem appeared in our first issue. If f has real coefficients and has zeros in $S = \{x + iy;\ y^2 \le x^2\}$ then does f' have the same property? The answer is no. For example take $f(x) = (x + 1)^3(x^2 - 2x + 2)$.

On the second question (where L and N are the zeros of the derivative of the cubic that has zeros at A, B and C) H. O. Davies contributes the remark that the incentre of the triangle ABC bisects each of the three angles LAN, LBN and LCN.

A LITTLE PUZZLE.

The question was whether it is obvious that a certain 4×4 determinant (det m in the notation below) is positive.

Using block notation put

$$a = \begin{bmatrix} x & y \\ y & -x \end{bmatrix} \qquad b = \begin{bmatrix} z & t \\ t & -z \end{bmatrix} \qquad m = \begin{bmatrix} a & b \\ b & -a \end{bmatrix}$$

R. B. Potts points out that

$$\begin{bmatrix} 1 & i \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ b & -a \end{bmatrix} \begin{bmatrix} 1 & i \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a + ib & 0 \\ b & -a + ib \end{bmatrix}$$

so that $\det m = \det(a + ib)\det(-a + ib)$ and since both a and b are real 2×2 it follows that

$$\det m = |\det(a + ib)|^2 \ge 0.$$

In fact this approach gives more than the inequality. Since a and b have the same structure as m,

$$\det(a + ib)\det(-a + ib) = \Big((x + iz)^2 + (y + it)^2\Big)\Big((x - iz)^2 + (y - it)^2\Big)$$

$$\det m = \Big((x - t)^2 + (y + z)^2\Big)\Big((x + t)^2 + (y - z)^2\Big).$$

Personally I would speak harshly to a student who when asked to show that something was positive started by squaring the expression. However we must make exceptions for old friends. B. B. Newman points out that squaring the given matrix leads to

$$(\det m)^2 = \begin{vmatrix} p & 0 & 0 & q \\ 0 & p & -q & 0 \\ 0 & -q & p & 0 \\ q & 0 & 0 & p \end{vmatrix} = p^4 - 2p^2q^2 + q^4$$

where $p = x^2 + y^2 + z^2 + t^2$ and $q = 2xt - 2yz$. This shows that

$$\pm \det m = (p + q)(p - q) = \Big((x + t)^2 + (y - z)^2\Big)\Big((x - t)^2 + (y + z)^2\Big).$$

Now it is easy (for instance by putting $y = z = t = o$) to show that we must take the plus sign, so that det m is found as before.

J. B. Parker writes that the hamfisted approach of expanding the determinant is made easier by putting $x = r \cos \theta$, $y = r \sin \theta$, $z = s \cos \phi$ and $t = s \sin \phi$.

GEOMETERY MADE DIFFICULT.

C. F. Moppert sends the following problem. Construct by Euclidean methods a line joining two given points, if your ruler is not long enough to reach from one point to the other. As the problem is essentially projective an elegant solution will not need compasses.

TRISECTING ANGLES.

G. Szekeres writes that this construction for trisecting an angle by means of a hyperbola is old. There are references to it by Rouse Ball and by Tietze, and there is evidence that for Descartes this construction was one of the incentives to invent analytic geometry.

HISTORICAL NOTE

The following news item about our patron James Cook comes from the Sydney Sunday Telegraph for 26th October, 1975.

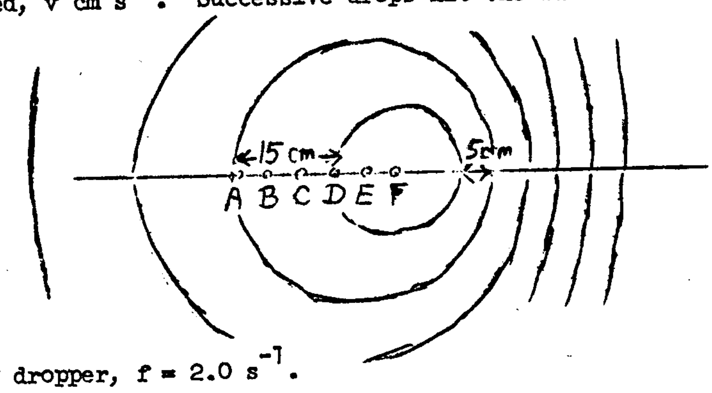
Captain Cook would have been 247 years old tomorrow. Looking down to-day on reprehensible Canberra he might have some reservations about having discovered Australia. But there are compensations. Two 30-storey towers of pensioner units are nearing completion in Sydney's Waterloo, the final stage of a Housing Commission project called Endeavour, after Cook's ship. Each floor will be named after something connected with the Captain's life. Floor five in one of the towers is to be called Free Love after Cook's first boat. Seven of the eight units are for single pensioners.

CONTRIBUTIONS.

Items for publication would be welcome. Send them to B. C. Rennie, Mathematics Department, James Cook University of North Queensland, Post Office 4811, North Queensland, Australia.

NEWS ON EDUCATION.

An eyedropper is used to release six drops into a still pool of water at a constant frequency, $f s^{-1}$, whilst the dropper is moving in a straight line at constant speed, v cm s^{-1}. Successive drops hit the water at distance a cm apart.



Frequency of dropper, $f = 2.0\ s^{-1}$.
Distance between successive drops, $a = 5.0$ cm.

What is the speed, v of the dropper?
Which of the points, A-F, represents the point of impact of the first drop?
Distances between successive waves to the left and right of point F are shown on the diagram. If the dropper had not moved, what would have been the wave length of the resulting wave pattern?
What is the speed of the waves shown in the diagram above?

Above is an extract from the Victorian Universities and Schools Examinations Board Higher School Certificate Examination, Physics, Tuesday 25th November, 1975 9.30 - 12.30.

The examiners evidently hold the common belief that every wave moves at a constant speed. The contrary opinion was asserted by Cauchy and Poisson soon after the time of James Cook (see H. Lamb's Hydrodynamics, page 431) and in fact it is clear from dimensional considerations that the circular waves from a sudden local disturbance start at zero speed and move outwards with constant acceleration, if the usual assumptions are made of no visocity, nosurface tension, infinite depth and infinitesimal waves.

In a book "Secret Naval Investigator" by Ashe Lincoln (William Kimber, London, 1961) there is told the story that in July 1942 the R.A.F. photographed a German minesweeper, the Sperrbrecher A1, detonating a British magnetic mine that had been laid in 11 fathoms near Lorient in the Bay of Biscay. The Navy was hopeful of finding out how far ahead of the Sperrbrecher the mine had exploded, for this would reveal the strength of the electromagnet that was mounted in the bows for clearing magnetic mines. The speed of the ship was known but the problem was to calculate the age of the explosion from the diameter of the circle of disturbance seen on the water. A sequence of photographs over 7½ seconds was available and it was suggested that from the increase in diameter in that time it would be possible (assuming constant wave speed) to estimate the time of the explosion. However the Lords of Admiralty unlike the VUSEB examiners did not trust the constant-speed theory, and so their calculations were based on experiments with real mines.