BIRATIONAL AND AFFINE GEOMETRY
MOSCOW, 23–27 APRIL 2012
ABSTRACTS

Ekaterina Amerik (Higher School of Economics, Moscow)
Remarks on self-maps with fixed points over a number field
Let $f : X \dashrightarrow X$ be a rational self-map with a fixed point q, where everything is defined over a number field K. We make some remarks on the dynamics of f in a p-adic neighbourhood of q for a suitable prime p. In particular we show that if the eigenvalues of Df_q are multiplicatively independent, then "most" algebraic points on X have Zariski-dense iterated orbits. (The starting motivation for this was an effort to find an easier proof of the potential density of the variety of lines on a cubic fourfold, due to Voisin and myself. If time permits, I shall also sketch this easier proof.) The talk is based on joint work with Bogomolov and Rovinsky.

Maxim Arap (John Hopkins University, Baltimore)
Classification of smooth weak Fano threefolds of Picard number two
Smooth Fano threefolds have been classified through the works of Fano, Iskovskikh, Shokurov, Mori and Mukai. Threefolds whose anti-canonical class is nef and big, but not ample, are called weak Fano (or almost Fano). The aim of this talk is to briefly review the classification of smooth Fano threefolds and to report the recent results on the classification of smooth weak Fano threefolds of Picard number two.

Jeremy Blanc (University of Basel)
Elements of finite order of the Cremona group
The Cremona group is the group of birational transformations of the plane. The study of its elements of finite order has a long history, and is now complete. I will explain how the classification is related to curves of fixed points, and explain that this geometric explanation does not easily generalise to finite groups or to higher dimension.

Serge Cantat (University of Rennes)
Linear Groups in Cremona Groups
I shall discuss the following question. Let G be a linear group, for example $\mathrm{GL}_n(k)$, where k is a field or the ring of integers. When does G embed into the Cremona group of birational transformations of a given variety M? Of course, this depends on G, on M, etc. Not much is known, but interesting problems show up.

Fabrizio Catanese (University of Bayreuth)
Inoue surfaces and Inoue type manifolds
Among minimal surfaces of general type with $p_g = 0$, there is only one class with $K^2 = 7$, discovered by Inoue 18 years ago. In recent work with Ingrid Bauer, we showed that these surfaces form an irreducible connected component of the moduli space, and that weak rigidity holds for them. Weak rigidity for X means, in its weaker form, that every other variety Y homotopically equivalent to X has the property that either Y or the conjugate variety belongs to an irreducible family containing X. We show this result by giving a different description of Inoue surfaces. This description lends itself to generalizations, which I will discuss during the talk. We define an Inoue type manifold as an ample divisor in a product of manifolds in the following list: Abelian varieties and quotients of irreducible locally symmetric spaces (including curves). Under some further assumptions, we can show semirigidity and weak rigidity results for Inoue type manifolds.

Jungkai Chen (National Taiwan University)
Factoring birational maps in dimension three
Flips, flops, and divisorial contractions are the elementary birational maps in minimal model program. We will demonstrate a factorization of birational maps into simpler ones in dimension three.

Sergey Galkin (IPMU, Tokyo)
Minifolds
Minifold is a manifold with a full exceptional collection of dimension plus one objects in its bounded derived category of coherent sheaves. With Anton Mellit we develop a program that allows to classify all minifolds numerically. We classify the minifolds in dimension up to four.

Adrien Dobouloz (University of Bourgogne)
Complement of hyperplane sub-bundles in projective space bundles over the projective line
A famous and surprising result of Danilov and Gizatullin asserts that the isomorphy type of the complement of an ample section in a Hirzebruch surface depends only its self-intersection, in particular it depends neither on the ambient surface nor on the chosen section.

In this talk, we will consider natural higher dimensional analogues where one removes hyperplane sub-bundles in arbitrary projective space bundles over the projective line. We will show in particular that for ample such sub-bundles, the isomorphy type of the complement is again uniquely determined by their top self-intersection.

Hubert Flenner (University of Bochum)
Deformation equivalence of affine normal surfaces
Two varieties X_1, X_2 over $\mathbb{C}$ are called deformation equivalent, if there is a family $(\mathcal{X}(s))_{s\in S}$ over a connected base S such that $X_i = \mathcal{X}(s_i)$, $i = 1, 2$, for some points $s_1, s_2 \in S$. If for

the class of varieties considered there is a moduli space then the varieties belonging to a connected component of this moduli space form just deformation equivalent varieties.

In this talk we consider deformation equivalence for the class say $\mathcal{C}$ of affine normal surfaces, which admit an $\mathbb{A}^1$-fibration. A family of surfaces in $\mathcal{C}$ consists in a completable flat morphism $p : \mathcal{V} \to S$ such that every fiber is a surface in $\mathcal{C}$. Here the morphism p is called *completable* if it is the restriction of some proper flat map $\bar{p} : \bar{\mathcal{V}} \to S$ to an open subset $\mathcal{V} \subset \bar{\mathcal{V}}$ such that the boundary $\mathcal{D} = \bar{\mathcal{V}} \backslash \mathcal{V}$ is a family of normal crossing divisors with constant dual graph. We note that except for a few exceptional cases one cannot expect for this class a moduli space.

We characterize in this talk as to when two surfaces in $\mathcal{C}$ are deformation equivalent. This characterization is given in purely combinatorial terms using the extended divisor of a surface with a $\mathbb{C}_+$-action. (Joint with S. Kaliman and M. Zaidenberg)

Rajendra Gurjar (University of Pune)

Fundamental Group Of Some Genus-2 Fibrations And Applications

We will prove that given a genus-2 fibration $f : X \to C$ on a smooth projective surface X such that $b_1(X) = b_1(C) + 2$, the fundamental group of X is almost isomorphic to $\pi_1(C) \times \pi_1(E)$, where E is an elliptic curve. We will also verify the Shafarevich Conjecture on holomorphic convexity of the universal cover of surfaces X with genus-2 fibration $X \to C$ such that $b_1(X) > b_1(C)$.

This is joint work with Sagar Kolte.

Shulim Kaliman (University of Miami)

Flexible varieties and Automorphism groups

Given an affine algebraic variety X of dimension $n \geqslant 2$, we let $\mathrm{SAut}(X)$ denote the special automorphism group of X i.e., the subgroup of the full automorphism group $\mathrm{Aut}(X)$ generated by all one-parameter unipotent subgroups. We show that if $\mathrm{SAut}(X)$ is transitive on the smooth locus X_{reg} then it is infinitely transitive on X_{reg}. In turn, the transitivity is equivalent to the flexibility of X. The latter means that for every smooth point $x \in X_{reg}$ the tangent space T_xX is spanned by the velocity vectors at x of one-parameter unipotent subgroups of $\mathrm{Aut}(X)$. We provide also different variations and applications.

Anne-Sophie Kaloghiros (Imperial College, London)

Relations in the Sarkisov Program

The Sarkisov Program studies birational maps between varieties that are end products of the Minimal Model Program (MMP) on nonsingular uniruled varieties. If X and Y are terminal $\mathbb{Q}$-factorial projective varieties endowed with a structure of Mori fibre space, any birational map between them can be decomposed into a finite number of elementary Sarkisov links. This decomposition is not unique in general, and any two distinct decompositions define a relation in the Sarkisov Program. This paper shows that relations in the Sarkisov Program are generated by some elementary relations. Roughly speaking, elementary relations are the relations among the end products of the MMP of Z over W, for suitable Z and W with relative Picard rank 3.

Ming-chang Kang (National Taiwan University)
Noether's problem and unramified Brauer groups
Let k be any field, G be a finite group acting on the rational function field $k(x_g : g \in G)$ by $h \cdot x_g = x_{hg}$ for any $h, g \in G$. Define $k(G) = k(x_g : g \in G)^G$. Noether's problem asks whether $k(G)$ is rational (= purely transcendental) over k. It is known that, if $\mathbb{C}(G)$ is rational over $\mathbb{C}$, then $B_0(G) = 0$ where $B_0(G)$ is the unramified Brauer group of $\mathbb{C}(G)$ over $\mathbb{C}$ investigated by Saltman and Bogomolov. Bogomolov proves that for any prime number p, there is a p-group G of order p^6 such that $B_0(G)$ is non-trivial and therefore $\mathbb{C}(G)$ is not rational over $\mathbb{C}$. He also shows that, if G is a p-group of order p^5, then $B_0(G) = 0$. The latter result was disproved by Moravec for $p = 3, 5, 7$ by the computer computing. The case for groups of order 32 and 64 was solved by Chu, Hu, Kang, Kunyavskii and Prokhorov.

We will prove the following theorems. Theorem 1 (Hoshi, Kang and Kunyavskii). Let p be any odd prime number and G be a group of order p^5. If G belongs to the isoclinism family Φ_{10} in R. James's classification of groups of order p^5 ("Math. Comp. 34 (1980) 613–637"), then $B_0(G) \neq 0$; in particular, $\mathbb{C}(G)$ is not rational over $\mathbb{C}$. On the other hand, if G doesn't belong to the isoclinism family Φ_6 or Φ_{10}, then $B_0(G) = 0$. Theorem 2 (Chu, Hoshi, Hu and Kang). Let G be a group of order 243 with exponent e. Let k be a field containing a primitive e-th root of unity. Then the followings are equivalent, (i) $k(G)$ is rational over k, (ii) $B_0(G) = 0$, (iii) G is not isomorphic to $G(243, i)$ with $28 \leq i \leq 30$ where $G(243, i)$ is the GAP code number for the i-th group of order 243.

Mariusz Koras (University of Warsaw)
Singularities of Z-acyclic normal surfaces of general type
We show that a homology plane of general type has at worst a single cyclic quotient singular point. An example of such a surface with a singular point does exist. We also show that the automorphism group of a smooth contractible surface of general type is cyclic. It is a joint work with R.Gurjar, M.Miyanishi and P.Russell.

Hanspeter Kraft (University of Basel)
Automorphisms of Complex Affine n-Space, Old and New
The group $GA(n)$ of regular automorphisms of complexe affine n-space $\mathbb{A}^n$ is an important example of a so-called ind-group (or infinite dimensional algebraic group). Quite a lot is known in dimension $n = 2$ due to the structure of $GA(2)$ as an amalgamated product. E.g. the unipotent elements form a closed subset, and the conjugacy classes of semisimple elements are closed. In higher dimension, most of these questions are completely open and we have only very few answers. Starting from classical results we will report on new developments and formulate a number of interesting problems.

Alexander Kuznetsov (Steklov Mathematical Institute, Moscow)
Categorical resolutions of singularities
A categorical resolution of singularities of an algebraic variety Y is a triangulated category T with an adjoint pair of functors between $D(Y)$ (the derived category of quasicoherent sheaves

on Y) and T, such that the composition is the identity endofunctor of $D(Y)$. If $X \to Y$ is a usual resolution, the derived category $D(X)$ with pullback and pushforward functors is a categorical resolution *only* if Y has rational singularities. However, I will explain that even if Y has nonrational singularities, still one can construct a categorical resolution of $D(Y)$ by gluing derived categories of appropriate smooth algebraic varieties. This is a work in progress, joint with Valery Lunts.

Leonid Makar-Limanov (Wayne State University, Detroit)
Locally nilpotent derivations and the AK (ML) invariant
There are a lot of questions related to affine algebraic geometry which have very simple formulations but which, sometimes, are not easy to answer. In my talk I'll describe some technique which allowed to answer some of these questions and simplified proofs of several known results.

Masayoshi Miyanishi (Kwansei Gakuin University)
$\mathbb{A}^1_*$-fibrations on affine threefolds
This is a joint work with R.V. Gurjar, M. Koras, K. Masuda and P. Russell. In affine algebraic geometry, our knowledge on affine algebraic surfaces is fairly rich with various methods of studying them. Meanwhile, knowledge of affine threefolds is very limited. It is partly because strong geometric approaches are not available or still under development.

A possible geometric approach is to limit ourselves to the case where the affine threefolds in consideration have fibrations by surfaces or curves, say the affine plane $\mathbb{A}^2$ or the curves $\mathbb{A}^1$ and $\mathbb{A}^1_*$, or have group actions of $\mathbb{G}_a$ or $\mathbb{G}_m$ which give the quotient morphisms over affine surfaces. Here the symbol $\mathbb{A}^1_*$ signifies the affine line $\mathbb{A}^1$ minus one point, often written as $\mathbb{C}^*$.

The main theme of the present talk is $\mathbb{A}^1_*$-fibrations defined on affine threefolds. The difference between $\mathbb{A}^1_*$-fibration and the quotient morphism by a $\mathbb{G}_m$-action is more essential than in the case of an $\mathbb{A}^1$-fibration and the quotient morphism by a $\mathbb{G}_a$-action. We consider necessary (and partly sufficient) conditions using the types of singular fibers under which a given $\mathbb{A}^1_*$-fibration becomes the quotient morphism by a $\mathbb{G}_m$-action. The structure of such affine threefolds can be elucidated. We also consider homology (or $\mathbb{Q}$-homology, or contractible) threefolds with $\mathbb{A}^1_*$-fibrations and apply the results to these classes of threefolds.

For example, we observe flat $\mathbb{A}^1_*$-fibrations which are expected to be surjective, but this turns out to be not the case by an example of Winkelmann. This example gives also a quasi-finite endomorphism of $\mathbb{A}^2$ which is not surjective.

Karol Palka (UQAM and Polish Academy of Sciences)
On the rectifiability of rational plane curves
Let $\bar{E} \subseteq \mathbb{P}^2$ be a rational cuspidal curve defined over complex numbers. The Coolidge-Nagata conjecture states that such a curve is *rectifiable*, i.e. it can be transformed into a line by a birational automorphism of $\mathbb{P}^2$. We will prove some new results in this direction, showing in particular that the conjecture holds if $\bar{E}$ has more than four cusps.

Vladimir Popov (Steklov Mathematical Institute, Moscow)
Rational actions on affine spaces
The last three decades were marked by growing interest in problems on regular actions of algebraic groups on affine spaces. Some of them admit counterparts related to rational actions. The talk is aimed at discussion of this “rational” viewpoint.

Damiano Testa (University of Warwick)
Plane cubics and their flex lines
Special tangent lines to plane curves have been studied intensively: flexes of plane cubics simplify the construction of the group law on the cubic, bitangents to plane quartics can be used to find the lines on a smooth cubic surface, flexes and bitangents appear in the Plücker formulas, and so on. For example, a smooth plane cubic has nine flex lines (and no bitangents).

This talk is based on joint work with M. Pacini: we show how to reconstruct a smooth plane cubic from its nine flex lines. The reconstruction procedure works over any field of characteristic different from three and is completely uniform with a unique exception.

Ernest Vinberg (Moscow State University)
Moduli of quartic surfaces and automorphic forms on symmetric domains of type IV
Contrary to dimensions 1 and 2, almost nothing was known about the structure of algebras of automorphic forms on multidimensional symmetric domains of type IV. The only such result was obtained by J. Igusa (1962), who proved that some algebra of automorphic forms on the 3-dimensional symmetric domain of type IV is free, and found the degrees of its generators. In 2010, the speaker managed to obtain analogous results in dimensions $4, 5, 6, 7$, making use of the interpretation of the projective spectra of the considered algebras of automorphic forms as the moduli varieties of some classes of quartic surfaces. These results imply, in particular, that the corresponding arithmetic groups are generated by reflections.

On the other hand, the speaker recently proved an old conjecture of O. V. Shvartsman (1981) on singularities at infinity of arithmetic quotients of symmetric domains of type IV, which implies that the algebra of automorphic forms on the n-dimensional symmetric domain of type IV with respect to some arithmetic group may be free only if $n \leqslant 10$. Moreover, it seems that there are only finitely many arithmetic groups with this property.

Jaroslaw Wisnievsky (University of Warsaw)
Another view on Cox rings: Jaczewski's theorem revisited
I will report on a joint work with Oskar Kedzierski. A generalized Euler sequence over a complete normal variety X is the unique extension of the trivial bundle $V \otimes \mathcal{O}_X$ by the sheaf of differentials Ω_X, given by the inclusion of a linear space $V \subset \mathrm{Ext}^1(\mathcal{O}_X, \Omega_X)$. For Λ, a lattice of Cartier divisors, let R_Λ denote the corresponding sheaf associated to V spanned by the first Chern classes of divisors in Λ. We prove that any projective, smooth variety on which the bundle R_Λ splits into a direct sum of line bundles is toric. We describe the

bundle R_Λ in terms of the sheaf of differentials on the characteristic space of the Cox ring, provided it is finitely generated. Moreover, we relate the finiteness of the module of sections of R_Λ and of the Cox ring of Λ.

David Wright (Washington University, Saint Louise)

Amalgamations and Automorphism Groups

We review some facts about automorphism groups over polynomial rings. It is shown the the tame subgroup of the group of polynomials automorphisms of affine 3-space can be realized as the product of three subgroups, amalgamated along pairwise intersections, in a manner that generalizes the well-known amalgamated free product structure in dimension 2. The result follows from the defining relations for the tame subgroup given by U. U. Umirbaev.