

## **13th EUROPT Workshop on Advances in Continuous Optimization**

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<http://www.maths.ed.ac.uk/hall/EUROPT15>

# Abstracts

## Exact Separation of $k$ -Projection Polytope Constraints

Miguel F. Anjos

Cutting planes are often used as an efficient means to tighten continuous relaxations of mixed-integer optimization problems and are a vital component of branch-and-cut algorithms. A critical step of any cutting plane algorithm is to find valid inequalities, or cuts, that improve the current relaxation of the integer-constrained problem. The maximally violated valid inequality problem aims to find the most violated inequality from a given family of cuts.  $k$ -projection polytope constraints are a family of cuts that are based on an inner description of the cut polytope of size  $k$  and are applied to  $k \times k$  principal minors of a semidefinite optimization relaxation. We propose a bilevel second order cone optimization approach to solve the maximally violated  $k$ -projection polytope constraint problem. We reformulate the bilevel problem as a single-level mixed binary second order cone optimization problem that can be solved using off-the-shelf conic optimization software. Additionally we present two methods for improving the computational performance of our approach, namely lexicographical ordering and a reformulation with fewer binary variables. All of the proposed formulations are exact. Preliminary results show the computational time and number of branch-and-bound nodes required to solve small instances in the context of the max-cut problem.

# Global convergence of the Douglas–Rachford method for some nonconvex feasibility problems

Francisco J. Aragón Artacho\*

## Abstract

In recent times the Douglas–Rachford algorithm has been empirically observed to solve a variety of nonconvex feasibility problems including those of a combinatorial nature. For many of these problems current theory is not sufficient to explain this observed success and is mainly concerned with questions of local convergence. In this talk we show global behavior of the method for some nonconvex feasibility problems.

This is a joint work with Jonathan M. Borwein and Matthew K. Tam.

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Dear Committee of 13<sup>th</sup>EUROPT Workshop on Advances in Continuous Optimization:

Following, I propose a title and an abstract for a talk.

Title: Pseudoinvexity in continuous vector optimization

Author: Manuel Arana Jiménez

Abstract:

Mathematical programming usually requires the study of optimality conditions joint to the properties of the functions involved. In this regard, it is well known that convex functions play an important role to obtain optimal solutions from critical points. This convex framework has been relaxed throughout last years thanks to generalized convexity. Pseudoinvexity is a suitable property to ensure that a critical point is really an optimal solution or an efficient solution, depending on the dimension (scalar or vectorial) of the considered programming problem. Furthermore, for this last, we show that pseudoinvexity is the minimal property of the functions in vector optimization.

In the scalar case of continuous optimization, the candidates for solutions can be identified from critical points, so as in the vector case, where the classical efficient solutions and weakly efficient solutions are proposed as an extension of these. In a new step, efficiency and optimality conditions are generalized under orders induced by cones with similar technics in vector optimization, where recently new generalizations of pseudoinvexity have appeared with the expected properties, called C-efficiency and C-pseudoinvexity. That is, C-pseudoinvexity is necessary and sufficient for a C-critical point to be a C-efficient solution. New challenges are proposed.

Best regards,  
Manuel



# Deregulated electricity markets with thermal losses and production bounds: models and variational reformulation

Didier Aussel<sup>1</sup> Michal Cervinka<sup>2</sup> Matthieu Marechal<sup>3</sup>

## Abstract

A multi-leader-common-follower game formulation has been recently used by many authors to model deregulated electricity markets. In our work, we first propose a model for the case of electricity market with thermal losses on transmission and with production bounds, a situation for which we emphasize several formulations based on different types of revenue functions of producers. Focusing on a problem of one particular producer, we provide and justify an MPCC reformulation of the producer's problem. Applying the generalized differential calculus, the so-called M-stationarity conditions are derived for the reformulated electricity market model. Finally, verification of suitable constraint qualification that can be used to obtain first order necessary optimality conditions for the respective MPCCs are discussed.

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# Constrained Best Euclidean Distance Embedding On A Sphere: A Matrix Optimization Approach

Shuanghua Bai, Hou-Duo Qi, Naihua Xiu

Presenter: Shuanghua Bai

## Abstract:

Euclidean distance matrices (EDM) are matrices of squared distances between points. EDM appears in many high-profile applications, among which is the data representation on a low dimensional manifold in high dimensional space. This technique is significant in data visualisation and analysis. In this talk, I'm going to talk about a spherical EDM embedding, which is a problem of data representation on a sphere of unknown radius. This problem arises from various disciplines such as Statistics (spatial data representation), Psychology (constrained multidimensional scaling), and Computer Science (machine learning and pattern recognition).

The best representation often needs to minimize a distance function of the data on a sphere as well as to satisfy some Euclidean distance constraints. It is those spherical and Euclidean distance constraints that present an enormous challenge to the existing algorithms. We reformulate the problem as an Euclidean distance matrix optimization problem with a low rank constraint. We then propose an iterative algorithm that uses a quadratically convergent Newton-CG method at its each step. We study fundamental issues including constraint nondegeneracy and the nonsingularity of generalized Jacobian that ensure the quadratic convergence of the Newton method. We use some classic examples from the spherical multidimensional scaling to demonstrate the flexibility of the algorithm in incorporating various constraints. Circle fitting is also an interesting application that we would like to present.

# Newton polytopes of stably coercive polynomials and related coercivity concepts

Tomáš Bajbar<sup>1</sup> and Oliver Stein<sup>2</sup>

We introduce the stable coercivity property on  $\mathbb{R}^n$  for multivariate polynomials  $f \in \mathbb{R}[x]$  and analyze it in terms of their Newton polytopes. For so-called gem regular polynomials we characterize their stable coercivity via four conditions solely containing information about the geometry of the vertex set of the Newton polytope at infinity, as well as sign conditions on the corresponding polynomial coefficients. For gem regular polynomials, it turns out, that these four conditions are equivalent to a special global growth-type condition being satisfied. For all other polynomials, the so-called gem irregular polynomials, we introduce some sufficient conditions for stable coercivity based on those from the regular case. Finally we analyze the relationship between the stable coercivity and the order of coercivity.

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# Backward Penalty Schemes for Monotone Inclusion Problems

**Abstract, to be held on the 13<sup>th</sup> EUROPT Workshop on Advances in Continuous Optimization**

Sebastian Banert, joint work with Radu Ioan Boț

March 27, 2015

Based on a common article of the authors, we are concerned with solving monotone inclusion problems expressed by the sum of a set-valued maximally monotone operator with a single-valued maximally monotone one and the normal cone to the nonempty set of zeros of another set-valued maximally monotone operator. Depending on the nature of the single-valued operator, we propose two iterative penalty schemes, both addressing the set-valued operators via backward steps. The single-valued operator is evaluated via a single forward step if it is cocoercive, and via two forward steps if it is monotone and Lipschitz continuous. The latter situation represents the starting point for dealing with complexly structured monotone inclusion problems from algorithmic point of view.

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# BFGS preconditioners for the normal equations arising in the Interior Point solution of constrained optimization problems

L. Bergamaschi\*, and A. Martínez†

March 27, 2015

We propose a class of deflated preconditioners to address the problem of efficiently solving the systems of the normal equations (NE) arising at each step of the interior point method in constrained optimization. As well known, the condition number of these SPD linear systems grows asymptotically as the sequence of the Newton iterates approaches the solution. Iterative solution of such linear systems, which is mandatory for large scale problems, requires the design of *ad hoc* preconditioners [4].

To this aim we will apply to a given preconditioner  $P_0$ , computed for the NE matrix at the initial Newton point, the BFGS-like low rank update as analyzed and tested in [1] for a general nonlinear system of equations as well as in [2, 3] for solving the sequence of nearly-singular linear systems to compute the leftmost eigenpairs of large SPD matrices by the Inexact Newton's method. The bounded deterioration property of the sequence of preconditioned matrices, as proved in [3] for singular or ill-conditioned Jacobians, is expected to mitigate the ill-conditioning of the systems of the normal equations towards the interior point solution of the optimization problem.

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# Improving interior point methods with continued iteration

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## ABSTRACT

The predictor corrector interior point method have been widely used to determine an optimal solution of linear programming problems, due to its efficiency and fast convergence. This iterative method determines a direction solving two linear systems in each iteration with the same symmetric positive definite matrix. One approach used to solve these linear systems is the Cholesky factorization. However, for many problems this factorization has a high computational cost. Thus, we propose the continued iteration approach in order to reduce of total computational time that it needs to obtain a linear programming problem optimal solution. The continued iteration consists in determining a new direction. This new direction is combined with the predictor corrector direction already computed by method. With the new direction is possible that a largest step size may be given, allowing reduction of infeasibilities. Although there is an computational effort increase per iteration to compute the continued iteration, the expected reduction in the number of iterations, enables to decrease the total computational time. Numerical experiments of the predictor corrector method with multiple centrality corrections, combined with the continued iteration for medium to large scale problems are performed.

# Gap functions and descent methods for quasi-equilibria

Giancarlo Bigi and Mauro Passacantando

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In this talk we focus on the *quasi-equilibrium problem*

$$\text{find } x^* \in C(x^*) \quad \text{s.t.} \quad f(x^*, y) \geq 0, \quad \forall y \in C(x^*) \quad (QEP)$$

where the bifunction  $f : \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}$  satisfies the equilibrium condition  $f(x, x) = 0$  for any  $x \in \mathbb{R}^n$  and the constraints are given by a set-valued map  $C : \mathbb{R}^n \rightrightarrows \mathbb{R}^n$  such that the set  $C(x)$  is closed and convex for any  $x \in \mathbb{R}^n$ .

QEPs are modelled upon quasi-variational inequalities (shortly QVIs). Also generalized Nash equilibrium problems (shortly GNEPs) can be reformulated through (QEP) with the Nikaido-Isoda aggregate bifunction. It is also worth noting that (QEP) is a natural generalization of the so-called abstract equilibrium problem (shortly EP), i.e., the case in which the set-valued map  $C$  is constant. As EP subsumes optimization, multiobjective optimization, variational inequalities, fixed point and complementarity problems, Nash equilibria in noncooperative games and inverse optimization in a unique mathematical model, further “quasi” type models could be analysed through the QEP format beyond QVIs and GNEPs.

Unlike QVI and GNEP, the QEP format did not receive much attention. The goal of the paper is to reformulate (QEP) as an optimization problem through a suitable gap function and develop an ad-hoc descent algorithm, supposing that the set-valued map  $C$  can be described by constraining bifunctions.

Gap functions have been originally conceived for variational inequalities and later extended to EPs, QVIs, jointly convex GNEPs via the Nikaido-Isoda bifunction and generic GNEPs via QVI reformulations. Though descent type methods based on gap functions have been extensively developed for EPs, the analysis of gap functions for QVIs and GNEPs is focused on smoothness properties and error bounds while no descent algorithm is developed.

Indeed, the reformulation of (QEP) as an optimization problem brings some difficult issues in devising descent methods which are not met in the EP case: the gap function is not necessarily differentiable even though the equilibrium and the constraining bifunctions are differentiable; the feasible region is given by the fixed points of the set-valued constraining map  $C$  and is therefore more difficult to handle; the so-called stationarity property, which guarantees all the stationary points of the gap function to be actually global minimizers and therefore solutions of (QEP), requires monotonicity assumptions both on the equilibrium and constraining bifunctions. These issues are dealt with in the talk. After the gap function has been introduced and the reformulation of (QEP) as an optimization problem shown, the smoothness properties of the gap function are analysed; in particular, an upper estimate of its Clarke directional derivative is given, which provides a key tool in devising the descent method. Furthermore, classes of constraints which allow guaranteeing the stationarity property are identified. The convergence of the descent method is proved under standard assumptions, and finally error bounds are given, which guarantee that the sequence generated by the algorithm is bounded.

## New bounds for the cp-rank in copositive optimization

Speaker: Immanuel M. Bomze, ISOR, University of Vienna, Austria

Coauthors: Abraham Berman, Peter J.C. Dickinson, Florian Jarre, Werner Schachinger, Naomi Shaked-Monderer, Georg Still, Reinhard Ullrich

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In copositive optimization, it is essential to determine the minimal number of nonnegative vectors whose dyadic products form, summed up, a given completely positive matrix (indeed, one of these vectors necessarily must be a solution to the original problem). This matrix parameter is called cp-rank. Since long, it has been an open problem to determine the maximal possible cp-rank for any fixed order. Now we can refute a twenty years old conjecture and show that the known upper bounds are asymptotically equal to the lower ones.

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## **A Bilevel Approach to Determine New Energy Service Prices**

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In this work, we model and solve an energy pricing problem (EPP) for residential Demand Side Management. To shift part of the load from peak periods to off-peak ones, an energy provider offers new services to consumers. The objective of EPP is to determine prices of these new services to maximize the Energy provider's profit, defined as the difference between services revenues and production costs, explicitly taking into account the behaviour and the invoice of the consumers. The production cost is defined as an increasing function of consumption. Thus higher revenues for EDF may be reached with smaller prices if these prices induce consumption shifting from peak periods to off-peak periods.

One of the challenges of EPP is to properly model customers' behaviour when choosing energy services. We assume that customers minimize their disutility function. This is the sum of the monetary costs and unwillingness to change consumption habits. Of course the second term may differ according to the sets of customers. In fact two main factors influence domestic consumption: the type of appliances used (heating or not, cooler or not, ..) and the willingness of the customers to change their consumption. Thus, in our model, customers are divided into categories (regarding their contracts with the energy provider) and segments regarding their reluctance to change their behaviour.

We propose several bilevel models to represent the hierarchical decision making process between the energy provider (the leader) and the consumers (the follower). Both decision variables (resp. objective functions) of the leader and the follower are continuous (resp. bilinear). Through a linearization scheme, the bilevel models are reformulated as mixed integer models and solved using CPLEX or any other off-the-shelf solver. Numerical results are given and discussed.

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## Modularity maximization clustering with cohesion conditions

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joint work with

A. Costa, Singapore University of Technology and Design, and P.Hansen, GERAD, HEC Montréal

The study of systems composed of interacting components through their representation as graphs is attracting a growing attention in a wide variety of domains. A modular structure characterizes many of these systems, that means that they contain subgroups of entities sharing some common properties. Therefore, a topic of particular interest is the identification of modules, called clusters or communities. The problem can be formulated using mathematical programming, where the idea is to optimize a function expressing the quality of the clustering partition. The most studied of these functions is the so-called *modularity*. We consider modularity maximization, formulated as a mixed-integer quadratic optimization problem, that has a convex continuous relaxation.

We address the problem of combining different clustering criteria and the corresponding impact on the optimization problem. Defining a good clustering criterion is in fact difficult. On the one hand, quality functions to be optimized have been proposed, and on the other hand, properties to be satisfied by each cluster of a partition have been suggested. It has recently been observed that one of the best known such properties, i.e., the *weak condition* [Proc. Natl. Acad. Sci. USA, 2004] was not satisfied by one or more clusters in a partition which maximizes some of the best known criteria. We consider five cluster-defining conditions, that we call *cohesion conditions* (including the weak one). We then modify the modularity optimization problem to add these conditions, one at a time, as constraints. These can be expressed as linear constraints (applying, for one of the considered conditions, a Fortet's linearization). Then, the obtained optimization models are still mixed-integer quadratic optimization problems, that we solve by exact methods. We thus discuss the solution of the new models, that enable to understand the impact of cohesion conditions on modularity maximization.

# Outer limit of subdifferentials and calmness moduli in linear and nonlinear programming\*

M.J. Cánovas<sup>†</sup>   R. Henrion<sup>‡</sup>   M.A. López<sup>§</sup>   J. Parra<sup>†</sup>

## Abstract

With a common background and motivation, the main contributions of this talk are developed in two different directions. Firstly, we are concerned with functions which are the maximum of a finite amount of continuously differentiable functions of  $n$  real variables, paying attention to the case of polyhedral functions. For these max-functions, we obtain some results about outer limits of subdifferentials, which are applied to derive an upper bound for the calmness modulus of nonlinear systems. When confined to the convex case, in addition, a lower bound on this modulus is also obtained. Secondly, by means of a KKT index set approach, we are also able to provide a point-based formula for the calmness modulus of the argmin mapping of linear programming problems without any uniqueness assumption on the optimal set. This formula still provides a lower bound in linear semi-infinite programming. Illustrative examples are given.

**Key words.** Calmness, local error bounds, variational analysis, linear programming, argmin mapping.

**Mathematics Subject Classification:** 90C31, 49J53, 94K40, 90C25, 90C05.

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# Linear programming on graphs through star sets and star complements

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Consider a graph  $G$  with vertex set  $V(G)$  of cardinality  $n$  and adjacency matrix  $A_G$ . Let  $\lambda$  be an eigenvalue of  $A_G$  (simply called an eigenvalue of  $G$ ) with multiplicity  $q$ . A vertex subset  $X$  with cardinality  $q$  such that  $\lambda$  is not an eigenvalue of  $G - X$  is called a  $\lambda$ -star set of  $G$ , while  $G - X$  is called a *star complement* for  $\lambda$  in  $G$  [3]. A  $(\kappa, \tau)$ -regular set of  $G$  is a vertex subset  $S$  inducing a  $k$ -regular subgraph such that every vertex not in  $S$  has  $\tau$  neighbors in it. A graph  $G$  has a  $(\kappa, \tau)$ -regular set iff the linear system

$$(A_G - (\kappa - \tau)I)x = \tau\hat{e}, \quad (1)$$

where  $I$  is the identity matrix of order  $n$  and  $\hat{e}$  is the all one  $n$ -vector, has a  $0 - 1$  solution  $x$ . In such a case, this solution is the characteristic vector of a  $(\kappa, \tau)$ -regular set. There are many combinatorial problems in graphs which are equivalent to the recognition of a  $(\kappa, \tau)$ -regular set. For instance, a  $p$ -regular graph of order  $n$  is strongly regular with parameters  $(n, p, e, f)$  iff for every vertex  $v$  its neighborhood is  $(e, f)$ -regular in  $G - v$  [2], a graph is Hamiltonian iff its line graph has a  $(2, 4)$ -regular set inducing a connected graph [2], etc. When  $\kappa - \tau$  is not an eigenvalue of  $G$  to decide whether  $G$  has a  $(\kappa, \tau)$ -regular set is easy, otherwise this problem could be very hard. However, assuming that  $\kappa - \tau$  is an eigenvalue of  $G$  and  $X \subset V(G)$  is a  $(\kappa - \tau)$ -star set, the linear system (1) can be replaced by the reduced system formed by the  $n - |X|$  equations corresponding to the vertices in  $V(G) \setminus X$ . Furthermore, the subset of columns of this reduced system indexed by the vertex subset  $Y$  defines a basic matrix iff  $V(G) \setminus Y$  is a  $(\kappa - \tau)$ -star set [1]. Based on these results, we are able to produce a sequence of  $(\kappa - \tau)$ -star sets towards to a  $(\kappa - \tau)$ -star set  $Y^*$  such that the corresponding star complement has a  $(\kappa, \tau)$ -regular set. A combinatorial simplex like technique is then introduced for deciding whether a  $(\kappa, \tau)$ -regular set there exists and several related results are presented.

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Keywords: Graph spectra, star sets and star complements, linear programming.

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# Simplex-like sequential methods for a class of generalized fractional functions

Riccardo Cambini\* - Laura Carosi\* - Laura Martein\* - Ezat Valipour<sup>†</sup>

Keywords: Generalized fractional programming, Pseudoconcavity, Sequential methods

## Abstract

A sequential method for a class of generalized fractional programming problems is proposed; the analyzed objective function is the ratio of powers of affine functions and the feasible region is a polyhedron. Theoretical properties of the optimization problem are studied and the characterization of the maximal domains of pseudoconcavity is given. When the objective function is pseudoconcave in the feasible region, the proposed algorithm takes advantage of the nice optimization properties of pseudoconcave functions; the particular structure of the objective function allows to provide a simplex-like algorithm even when the objective function is not pseudoconcave. To attest the effectiveness of the proposed techniques, the results of a computational experience are then presented.

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## **Inversion, history matching, clustering and linear algebra**

Andrew Conn

The inversion of large-scale ill-posed problems introduces multiple challenges, including identifying appropriate noise model, prescription of a suitable prior information, incorporation of heterogeneous sources of data, and definition of an appropriate optimization scheme. In this study, the inherent uncertainty of the problem is mitigated by devising efficient and comprehensive approaches for prior sampling. In particular, geo-statisticians may often propose large sets of prior samples that regardless of their apparent geological distinction are almost entirely flow equivalent. As an antidote, a reduced space hierarchical clustering of flow relevant indicators is proposed for aggregation of these samples. The effectiveness of the method is demonstrated both with synthetic and field scale data. In addition, numerical linear algebra techniques that exploit the special structure of the underlying problems are elucidated.

## **Theoretical and Practical Convergence of a Self-Adaptive Penalty Firefly Algorithm for Constrained Global Optimization**

M. Fernanda P. Costa, Rogério B. Francisco, Ana Maria A.C. Rocha and Edite M.G.P. Fernandes

In this paper, we propose a self-adaptive penalty function and present a penalty-based algorithm for solving constrained global optimization problems. We prove that the general constrained optimization problem is equivalent to a bound constrained problem in the sense that they have the same global solutions. Some properties regarding the convergence of the penalty-based algorithm are established. The global minimizer of the penalty function, subject to a set of bound constraints, is obtained by the firefly algorithm (FA), a swarm intelligence method inspired by the social behavior of fireflies and based on their flashing and attraction characteristics. The FA is hybridized with a local direct search procedure aiming to enhance the quality of the obtained solutions. Numerical experiments using a set of benchmark global optimization problems are carried out to analyze the performance of the proposed algorithm. A comparison with other penalty-based approaches shows that the algorithm is effective in reaching the global solutions.



# A Black-Box algorithm to solve Simulation-Optimization problems with chance constraints: general framework and applications

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## Abstract

Many real world applications lack of reliable analytic models for optimizing the system performance. Even if it is possible to design an analytical model for the problem, this may result either too complex to be managed efficiently by the decision maker or too simple and therefore some complex dynamics of the system can be neglected. Moreover, when stochastic variables are involved in the complex system, their distribution should be known a priori in order to correctly define an analytic model. In many applications, simulation-based approaches are superior to analytic models for investigating complex stochastic systems. Indeed, the system performance can be described by means of a black-box function and there is no need to know the probability distribution of the stochastic components. Moreover, this distribution can change accordingly to the system design variables and stochastic constraints can also be included in the model, thus adding an higher level of uncertainty.

Our framework consider optimization problems where the objective is a black-box function which depends both on the design variables and on the stochastic components which distribution in turn depends on the design variable. There are also some constraints on the design and stochastic variables. In particular in this work we propose an approach that tackles the high computational cost, the black-box formulation and the stochasticity. In recent literature, these features have been addressed separately and, as in stochastic optimization, there is the underline assumption that the probability distribution of the stochastic components do not depend on the design variables.

The optimization algorithm used is based on a multi-start approach to explore the feasible region for the design variable. Then a sample of the stochastic component is generated and its size is adapted dynamically according to a defined criterion which aims to guarantee feasibility in probability. In other words, we define a confidence level to accept a point as feasible and then the sample size is accordingly adapted (it can be different by each design variable). In this way we can control the feasibility and the computational cost of evaluating the model, which is related to the sampling cost.

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# **Fluence Map Non-Linear Continuous Optimization for IMRT Treatment Planning**

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Radiation therapy is one of the treatments used for cancer patients. Its aim is to destroy cancer cells through radiation, but at the same time spare healthy tissue that can also be damaged by radiation. Intensity Modulated Radiation Therapy (IMRT) is one type of radiation therapy where it is possible to modulate the radiation intensities that are delivered to the patient from each radiation incidence. One of the problems that has to be solved during treatment planning is to find the best possible intensity profiles (fluence maps) for each radiation direction (fluence map optimization – FMO). This is usually done by resorting to an optimization programming model, which requires the treatment planner to define several different parameters (like weights and lower/upper bounds). The tuning of these parameters is usually done by resorting to a lengthy trial-and-error procedure: the planner tries a set of parameters, solves the optimization problem and performs dosimetric calculations to check whether the current solution is complying with the medical prescription. If it is not, then he changes the model's parameters, and tries again.

We propose a completely different methodological approach for FMO. A non-linear unconstrained continuous programming model will be used, and will be iteratively solved by having the model's parameters changed in an automated way using a fuzzy reasoning inference system. This methodology releases the human planner from trial-and-error procedures, being a completely automated approach for FMO. The planner is only asked to define the constraints that should be satisfied so that a treatment plan is considered admissible. The proposed methodology is able of calculating high quality solutions in reasonable computational times. Computational results using retrospective treated head-and-neck cancer patients will be shown.

*Title:*           **Considering Copositivity Locally**

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*Session:*       Gabriele Eichfelder, “Copositive Optimization”.

*Conference:*   13th EUROPT Workshop  
on Advances in Continuous Optimization

Abstract:

In this talk we characterise the cone of feasible directions for a copositive matrix  $A \in \mathcal{COP}^n$ , i.e., the convex cone of symmetric matrices  $B$  such that there exists  $\delta > 0$  satisfying  $A + \delta B \in \mathcal{COP}^n$ . This furnishes characterisations of the tangent cone, the minimal face and the minimal exposed face for  $A$  in  $\mathcal{COP}^n$ . All of the characterisations are in the form of sets of linear inequalities constructed from the (minimal) zeros of  $A$ .

# Weak subgradient method in unconstrained optimization

Gulcin Dinc Yalcin, Refail Kasimbeyli

In this work we study a new subgradient method for solving unconstrained optimization problems. The method uses weak subgradients of the objective function at every iteration. Due to this reason the weak subgradient method does not require convexity and uses dynamic step sizes. Convergence properties for the sequence of solutions generated by iterations of the proposed algorithm are investigated.

Keywords: weak subgradient, nonsmooth optimization, nonconvex programming, continuous optimization

## The update of sequences of some incomplete decompositions matrices for preconditioning

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Simulation with models based on partial differential equations require very often the solution of (sequences of) large and sparse algebraic linear systems. In multidimensional domains, *preconditioned* Krylov iterative solvers are often appropriate for these duties. Therefore, the search for efficient preconditioners for Krylov subspace methods is a crucial theme.

Recent developments, especially in computing hardware, have renewed the interest in *approximate inverse preconditioners* in factorized form, because their application during the solution process can be more efficient.

We present some ideas for updating approximate inverse preconditioners in factorized form. Computational costs, reorderings and implementation issues are considered.

## **Robust Nonlinear Dynamic Parameter Identification using Decomposition of Nonlinearities**

Dr.-Ing. Mitja Echim (Universität Bremen)

For the optimization of a given application a realistic model is needed. In many applications a system of ordinary nonlinear differential equations is used to describe the dynamic behavior of a given situation. To increase accuracy in general these models depend on some parameters (e.g. efficiency coefficient, capacities or diameters) which have to be adapted concerning some measurements. This task can be done automatically with mathematical optimization techniques (e.g. sequential quadratic programming).

Using the conventional approach (solve the ode system numerically in every optimization iteration to calculate the value of the objective function) the resulting objective function is highly nonlinear. In order to receive a good fit with given measurements a very good initial guess for the parameters is required. The very high sensitivity of the objective function regarding small changes of the parameters in nonlinear ode systems causes many difficulties during the optimization process.

In this work the concept of decomposition of nonlinearities is used to design a robust and efficient algorithm to solve nonlinear dynamic parameter identification problems. The integration of the ode systems is proposed to be performed within the optimization process which leads to a high dimensional optimization problem with many constraints (depending on the dimension of the given data). Using the sparse SQP method WORHP the solution of the large scaled optimization problem can be done efficiently.

The benefits and the latest results of this technique will be illustrated by an application of a turbocharger design within a framework with more than 2 million variables and constraints.

Abstract:

## Copositivity tests based on the Linear Complementarity Problem

Gabriele Eichfelder<sup>1</sup>

A symmetric  $n \times n$  matrix is called copositive if it generates a quadratic form which takes no negative values on the nonnegative orthant. The problem of minimizing a linear form over the copositive cone, i. e. over the cone of copositive matrices, is called a copositive optimization problem. This type of optimization problems, and thus the possibility to evaluate whether a matrix is copositive, is of interest due to its relation to combinatorial optimization. For instance the Maximum Clique Problem can be re-formulated as a copositive optimization problem. It was even shown that every quadratic program with linear constraints can be formulated as a copositive program, also if some of the variables are binary.

Various authors have proposed copositivity tests in the literature, but there are only a few implemented numerical algorithms which apply to general symmetric matrices without any structural assumptions or dimensional restrictions and which are not just recursive, i.e., do not rely on information taken from all principal submatrices. In this talk we present new copositivity tests which are of this type.

The tests are based on new necessary and sufficient conditions requiring the solution of linear complementarity problems (LCP). Methodologies involving Lemke's method, an enumerative algorithm and a linear mixed-integer programming formulation are proposed to solve the required LCPs. A new necessary condition for (strict) copositivity based on solving a Linear Program (LP) is also discussed, which can be used as a preprocessing step. The algorithms are applied to test matrices from the literature and to max-clique instances with matrices up to dimension  $496 \times 496$ . We compare the results with those from three other copositivity tests from the literature as well as with a general global optimization solver. The numerical results are very promising and equally good and in many cases better than the results reported elsewhere.

This is a joint work with Carmo Brás (Faculdade de Ciências e Tecnologia, Universidade Nova de Lisboa, Portugal) and Joaquim Júdice (Instituto de Telecomunicações, Portugal).

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# Minimization in Banach spaces by Conjugate Gradient method

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In many inverse problems, the “geometry” of the  $L^p$  Banach spaces, for  $1 < p < 2$ , can substantially reduce the over-smoothness effects with respect to any conventional iterative regularization algorithm in the  $L^2$  Hilbert space [1].

In this work we introduce a generalization of the conjugate gradient method for the minimization of the  $p$ -norm cost functional  $\Phi(x) = \|Ax - y\|_Y^p$ , related to the solution of the ill-posed operator equation  $Ax = y$ , where  $A : X \rightarrow Y$  is a linear operator between two Banach spaces. We first prove the convergence of the iterations to a solution of the operator equation in both noise-free and noisy data cases [2]. Then we show that the high convergence speed of conventional conjugate gradient in  $L^2$  Hilbert space gives rise to a fast iterative method in  $L^p$  spaces too.

The minimization algorithm is applied to enhance the spatial resolution of microwave radiometer data [3]. The problem which describes the relationship between the coarse but partially correlated measurements and the brightness temperature is ill-posed problem since it belongs to the class of Fredholm integral equation of the first kind.

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# Global Mixed Integer Nonlinear Optimization by Metaheuristic Techniques

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## Abstract

Greedy Randomized Adaptive Search Procedure (GRASP), originally proposed by Feo and Resende (1989), is a multi-start metaheuristic for combinatorial problems, in which each iteration consists of two phases: construction and local search. The construction phase builds a feasible solution, whose neighborhood is investigated until a local minimum is found during the local search phase. The best overall solution is kept as the solution. Continuous GRASP (C-GRASP), proposed by Hirsh, Meneses, Pardalos, and Resende (2006) extends GRASP from the domain of discrete optimization to that of continuous global optimization to address continuous problems subject to box constraints. C-GRASP has been very successful on a wide number of applications.

Facó, Resende, and Silva (2011, 2012, 2013) proposed new versions of C-GRASP able to incorporate more general linear and nonlinear constraints. For general linear constraints we can eliminate some variables of the linearly constrained problem, and solve a reduced problem with only the box constraints by C-GRASP. General nonlinear constraints can be penalized in the objective-function with the addition of quadratic terms. Numerical experiments to find global solutions for constrained NLP continuous problems have obtained encouraging results.

Here we consider Global Mixed Integer Nonlinear Programming problems. Instead of relaxing the discrete variables, and rounding the continuous solution by branch-and-bound or cutting planes procedures that may suffer the effects of the curse of dimensionality, we propose a different approach. A new GRASP version considering both - discrete and continuous variables - is presented using an a priori choice of the discrete variables. GRASP random search and local improvement phases use simultaneously a discrete and a continuous set. We present difficult MINLP problems that have been solved with this approach.

# **Stable Strong Fenchel and Lagrange duality for evenly convex optimization problems**

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**Abstract:** By means of a conjugation scheme based on generalized convex conjugation theory instead of Fenchel conjugation, we build an alternative dual problem, using the perturbational approach, for a general optimization primal one defined on a separated locally convex topological space. Conditions guaranteeing strong duality for disturbed primal problems by continuous linear functionals and their respective dual problems, which is named stable strong duality, are established. In these conditions, the evenly convexity of the perturbation function will play a fundamental role. Stable strong duality will also be studied in particular for Fenchel and Lagrange primal-dual problems, obtaining moreover a characterization for Fenchel case.

# A tri-objective model for locating a semi-desirable facility in the plane

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## Abstract

We consider the problem of locating a single semi-desirable facility in the plane. Three objectives are taken into consideration. The first one maximizes the efficiency of the service provided by the facility to some users, by minimizing the sum of weighted distances between the facility and those users. The second one minimizes the social cost caused by the undesirable effects produced by the facility, by minimizing the sum of the repulsions of the affected people (as they feel it). The third one aims to distribute the repulsions fairly (as equal as possible) among the affected people. Two recent general-purpose multi-objective evolutionary algorithms, MOEA/D and FEMOEA, are suggested to obtain a discrete approximation of its Pareto-front. A computational study shows that both algorithms are suitable to cope with the problem, although FEMOEA seems to obtain slightly better results, especially for larger instances.

**Keywords:** Semi-desirable facility, Nonlinear tri-objective optimization problem, Gini index, Pareto-front, Evolutionary algorithms

# A COMPARATIVE STUDY OF RELAXATION ALGORITHMS FOR THE LINEAR SEMI-INFINITE FEASIBILITY PROBLEM

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**Abstract:** The feasibility problem of finding a feasible solution to a linear semi-infinite system arises in different contexts. This talk proposes a new relaxation algorithm with variable step size which uses the Extended Cutting Angle Method (ECAM) for solving the global optimization subproblems. The algorithm converges linearly to a feasible solution under mild conditions. Numerical experiments show that it can compete successfully, from the computational efficiency point of view, with the already known fixed step size relaxation algorithm.

**Keywords:** Linear semi-infinite systems, feasibility problem, relaxation method, cutting angle method.

## Augmented Lagrangians, non-negative QP and extensions

**Roger Fletcher** (*University of Dundee*)

Augmented Lagrangians are not commonly used to solve quadratic programming (QP) problems with inequality constraints due to second derivative discontinuities in both the inner and outer iterations. For non-negative QP (that is the only constraints on the variables are  $x \geq 0$ ) a transformation of variables is described that removes these discontinuities from the inner iteration which can then be solved simply as a system of linear equations. The outer iteration objective function retains second derivative discontinuities but under mild assumptions is strictly convex, and can, for example, be minimized quickly and efficiently by Newton's method with a suitable line search. The method avoids the combinatorial difficulties associated with active set methods and convergence is guaranteed. Extension of these ideas to include  $m$  equality constraints on the variables is described. These are particularly effective in the case  $m = 1$ . Inverse problems provide a rich source of these problem types, and numerical examples of these are described.

# A problem generator and performance of methods for big data optimization

Kimon Fountoulakis and Jacek Gondzio

## Abstract

We present an instance generator for problems:

$$\text{minimize } f_\tau(x) := \tau \|x\|_1 + \frac{1}{2} \|Ax - b\|_2^2, \quad (1)$$

where  $A \in \mathbb{R}^{m \times n}$ ,  $b \in \mathbb{R}^m$  and  $\tau > 0$ . The generator is inspired by the one presented in Section 6 of [2], for details see [1]. The advantage of our modified version is that it allows to control the properties of matrix  $A$  and the optimal solution  $\tilde{x}$  of (1). For example, the sparsity of matrix  $A$ , its spectral decomposition, the sparsity and the norm of  $\tilde{x}$ . The generator has very low memory requirements and scales well with the dimensions of the problem. We believe that the flexibility of the proposed generator will cover the need for generation of various good test problems.

Using the proposed generator we study the performance of first- and second-order optimization methods as the conditioning and the dimensions of the problem increase **up to one trillion**.

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# Preferences in Mean-Risk Portfolio Optimization

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## **Abstract:**

Solving the portfolio selection problem relies on models for preference between random variables representing portfolio returns. Choosing a specific model is itself a problem because each type of models assumes a different vision on choice under risk, different theoretical basis with strengths and weaknesses, and different degrees of computational tractability. In this paper, we present a new *Mean-Risk* model of portfolio selection. Its novelty consists in using of a new risk measure based on the modified loss distribution function.

The contribution of this paper is twofold.

Firstly, we propose a risk measure defined using the modified loss distribution according to the investor's risk and loss aversion preferences and establish its properties. A portfolio selection model in the *Mean-Risk* framework using the risk measure previously defined is proposed. Equivalent formulations of the model generating the same efficient frontier are given.

Secondly, the advantages of the new approach are investigated using real world data from New York Stock Exchange. The differences and similarities between the efficient frontier of the proposed model and the classical *Mean - Variance* and *Mean - Conditional Value at Risk* frontiers are quantified and interpreted. The investor's benefits of using the proposed model are assessed: out-of-sample experiments show that the efficient portfolios obtained by using the new model exhibit lower risk levels (at the same return levels) and an increased satisfaction compared to the classical *Mean-Risk* models.

# Memory-efficient interior point method for solving a time-dependent scheduling problem

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We propose a new algorithm, based on the interior point method [3], for approximate solving the following open time-dependent scheduling problem  $(P)$ , [1,4]. A set of  $n$  independent non-preemptable jobs,  $J_1, J_2, \dots, J_n$ , has to be scheduled on a single machine. The processing time  $p_j$  of  $J_j$  linearly deteriorates in time, i.e.  $p_j = 1 + \alpha_j t$ , where deterioration rate  $\alpha_j > 0$  and the job starting time  $t \geq 0$  for  $1 \leq j \leq n$ . The aim is to find a schedule with minimal sum  $\sum_{j=1}^n C_{[j]}$ , where  $C_{[j]}$  is the completion time of the  $j$ th job in the schedule.

The main idea of our algorithm is as follows. Let  $C(a)$  be the vector of job completion times,  $A(a)$  be an  $n \times n$  square matrix with 1's on the main diagonal, components  $a_j = 1 + \alpha_j$  of the sequence  $a$  multiplied by  $-1$  below the main diagonal and equal to 0 otherwise, and  $d = (1, 1, \dots, 1)^\top$ , [1]. Then problem  $(P)$  is  $\min W_P(a) := \|C(a)\|_1$  subject to  $A(a)C(a) = d$ , where the minimization of  $W_P(a)$  is taken over all  $a$  from the set  $\mathcal{P}(a^\circ)$  of all permutations of an initial sequence  $a^\circ$  of deterioration coefficients  $a_j = 1 + \alpha_j$  for  $1 \leq j \leq n$ .

Since any optimal solution to  $(P)$  must be a V-shaped sequence, we consider the polyhedron of all  $2^n$  such V-sequences for a given  $a^\circ$ . Next, we attach to each vertex of this polyhedron a permutation matrix of a special kind and consider the convex hull of these permutation matrices, which coincides with  $n$ -dimensional polyhedron of all doubly stochastic matrices of a special kind. The convex polyhedron, in turn, we use in a new formulation of problem  $(P)$ , to which we apply the primal-dual interior point method.

In order to make our algorithm computationally more efficient compared to its predecessor [2], we propose to replace in interior point method the Newton method by an another method, preserving the same size of matrices but without using the Hessian inverse. We also propose to reduce memory usage by applying a steepest descent method to the goal function with the barrier and the barrier and penalty components added, respectively. We illustrate the presentation of the proposed algorithm by results of extensive numerical experiments.

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# Complete Characterizations of Tilt Stability in Nonlinear Programming under Weakest Qualification Conditions

Helmut Gfrerer\*

and

Boris S. Mordukhovich<sup>†</sup>(coauthor)

This talk is devoted to the study of tilt stability of local minimizers for classical nonlinear programs with equality and inequality constraints in finite dimensions described by twice continuously differentiable functions. The importance of tilt stability has been well recognized from both theoretical and numerical perspectives of optimization, and this area of research has drawn much attention in the literature, especially in recent years. Based on advanced techniques of variational analysis and generalized differentiation, we derive here complete pointbased second-order characterizations of tilt-stable minimizers entirely in terms of the initial program data under the new qualification conditions, which are the weakest ones for the study of tilt stability.

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**Speaker:** Miguel A. Goberna, University of Alicante (Spain)

**Title:** Radii of robust feasibility and robust optimality for uncertain convex programs

**Abstract**

In this talk we suppose that the data (the objective and the constraint functions) of a given *nominal* scalar convex program

$$\bar{P} : \min \{ \bar{f}(x) : \bar{g}_j(x) \leq 0, j = 1, \dots, m \},$$

may suffer linear and affine perturbations, respectively, giving rise to perturbed functions of the form

$$f(x) = \bar{f}(x) + c^\top x, \text{ with } c \in \gamma \mathbb{B}_n,$$

and

$$g_j(x) = \bar{g}_j(x) + a_j^\top x + b_j, \text{ with } (a_j, b_j) \in \alpha_j \mathbb{B}_{n+1}, j = 1, \dots, m,$$

where  $\gamma, \alpha_1, \dots, \alpha_m$  are non-negative parameters to be chosen by the decision maker (the closed balls  $\gamma \mathbb{B}_n, \alpha_1 \mathbb{B}_{n+1}, \dots, \alpha_m \mathbb{B}_{n+1}$  are called *uncertainty sets*).

If  $\alpha_1, \dots, \alpha_m$  are taken too large, the *robust feasible set*

$$X(\alpha_1, \dots, \alpha_m) := \{x \in \mathbb{R}^n : \bar{g}_j(x) + a_j^\top x + b_j \leq 0, \forall (a_j, b_j) \in \alpha_j \mathbb{B}_{n+1}, j = 1, \dots, m\}$$

could be empty. In order to guarantee the feasibility under any possible perturbation we define the *radius of robust feasibility* as the non-negative number

$$\sup \{ \min \{ \alpha_1, \dots, \alpha_m \} : X(\alpha_1, \dots, \alpha_m) \neq \emptyset \}.$$

Now assume that  $\alpha_1, \dots, \alpha_m$  have been chosen so that  $X(\alpha_1, \dots, \alpha_m) \neq \emptyset$  and let  $\gamma > 0$  be given. A robust feasible solution  $\bar{x}$  is called *highly robust optimal solution* when it is an optimal solution of the parametric convex problem

$$P(c) : \min \{ \bar{f}(x) + c^\top x : x \in X(\alpha_1, \dots, \alpha_m) \}$$

for any  $c \in \gamma \mathbb{B}_n$ , which implies that  $\bar{x}$  is the unique optimal solution to

$$\min \{ \bar{f}(x) : x \in X(\alpha_1, \dots, \alpha_m) \}.$$

In order to guarantee the existence of highly robust optimal solution we introduce the *radius of optimality* as

$$\sup \{ \gamma \in \mathbb{R}_+ : \bar{x} \in \operatorname{argmin} P(c) \ \forall c \in \gamma \mathbb{B}_n \}.$$

In [1], we give exact formulas for the radii of robust feasibility and robust optimality of uncertain convex programs.

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# ADVANCES IN SOME FRACTIONAL VARIATIONAL PROBLEMS WITH CAPUTO DERIVATIVES

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Derivatives and integrals of fractional order have recently gained more attention due to their successful application to non local phenomena. Motivated by numerous applications in physics and other scientific areas, fractional calculus of variations finds itself in fast development.

In this work we consider variational problems of the following kind: find  $y \in {}^{\alpha}E_b^{\beta}$ , so that it maximizes or minimizes the functional

$$J(y) = \int_a^b L(x, y, {}^C D_x^{\alpha} y, {}^C D_b^{\beta} y) dx$$

where  $L$  is a Langrangian function,  ${}^C D_x^{\alpha} y$ ,  ${}^C D_b^{\beta} y$  denote the left and right fractional derivatives in Caputo's sense of order  $\alpha, \beta \in (0, 1)$  and

$${}^{\alpha}E_b^{\beta} = \{y : [a, b] \rightarrow \mathbb{R} : {}^C D_x^{\alpha} y, {}^C D_b^{\beta} y \text{ exist and are continuous on } [a, b]\}$$

Different necessary and sufficient conditions of optimality are considered, particularly some of fractional Euler-Lagrange equation type:

$$\frac{\partial L}{\partial y} + {}^C D_b^{\alpha} \frac{\partial L}{\partial {}^C D_x^{\alpha} y} + {}^C D_x^{\beta} \frac{\partial L}{\partial {}^C D_b^{\beta} y} = 0.$$

Several fractional variational problems are presented: with fixed or free boundary conditions, in presence of integral constraints that also depend on Caputo derivatives and some examples are given.

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## **Preconditioning sequences of linear systems with generalizations of quasi-Newton formulas**

Robert Gower

Abstract:

At the heart of Newton based optimization methods is a sequence of symmetric linear systems. Each consecutive system in this sequence is similar to the next, so solving them separately is a waste of computational effort. Here we describe automatic preconditioning techniques for iterative methods for solving such sequences of systems by maintaining an estimate of the inverse system matrix. We update the estimate of the inverse system matrix with quasi-Newton type formulas based on what we call an action constraint instead of the secant equation.

## About closedness type regularity conditions in convex optimization

Sorin-Mihai Grad, Chemnitz University of Technology

The closedness type regularity conditions have proven during the last decade to be viable alternatives to their more restrictive interiority type counterparts, in both convex optimization and different areas where it was successfully applied, from which we mention only variational inequalities and the theory of monotone operators.

In this talk we will de- and reconstruct some general closedness type regularity conditions, in order to stress that they arise naturally when dealing with optimization problems, as noticed earlier by Precupanu.

To this end, we characterize first (stable)  $\epsilon$ -duality for general optimization problems formulated by means of perturbation functions by making use of epigraphs of the conjugates and (convex) subdifferentials of the involved functions.

These results are used in order to de- and reconstruct recent closedness type regularity conditions from the literature that guarantee (stable) zero duality gap and (stable) strong duality in convex optimization, respectively.

The general results are then particularized for constrained and unconstrained optimization problems as well as for composed convex optimization problems and different results of the author and his coauthors or due to Bo\c{t} and Wanka, and Burachik, Jeyakumar and Wu, respectively, are rediscovered as special cases.

## **Multilevel algorithms for large scale nonlinear optimization problems.**

Serge Gratton, University of Toulouse, CERFACS-IRIT joint laboratory

Multidomain and multigrid methods are well-known techniques for obtaining efficient solution methods that are mostly used for solving quadratic problems. They have been studied for decades, have been implemented in well-known software and are nowadays used routinely for solving industrial large scale problems on modern parallel computers.

The purpose of this talk is to show ways to make use of these exceptional properties to obtain general solvers for large scale non-convex optimization problems. An important issue will be the design of a suitable technique for obtaining convergence irrespective of the starting point. We will show that this can be done without compromising the performance of the underlying multilevel technique. The resulting algorithms not only enjoy satisfactory worst case complexity bounds, but are also very efficient on several problems arising for instance in the calculus of variations.

We conclude with specialized solvers in the case of large scale nonlinear least-squares problem arising in geophysics. We show that the structure of the problem can be used to further enhance the performance of our methods. We are able in this case to design efficient dual solvers that also open the door to grid adaptivity. The efficiency of the method will be demonstrated on inverse problems.

## **Tangent cones of inverse Images of semi-algebraic sets**

Dominik Dorsch and Harald Günzel

Abstract: We consider feasible sets that are defined as inverse images of an arbitrary but fixed semi-algebraic set. For such sets we define appropriate versions of tangent cones. For generic problem data we show that the tangent cones are pullbacks of tangent cones of the given semi-algebraic set. We also show that there exists a stratification of the given semi-algebraic set such that the structure of the tangent cones is in some sense stable on each stratum.

# 13th EUROPT Workshop on Advances in Continuous Optimization

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## Title

FLO – A tool for solving multi-objective location problems

## Abstract

Project FLO (Facility Location Optimizer) aims at providing a MATLAB-based tool for solving multi-objective as well as single-objective location problems. We will present algorithms and features of the current version of the Software FLO. The classical single-facility multi-objective location problem consists in minimizing the distances between a new facility and a finite number of given facilities in the plane simultaneously. In this talk, we focus on location problems with uncertainties in the data and present results concerning computing the set of robust efficient solutions for special classes of multi-objective location problems with uncertainties in the data. At the end of the talk, we give an overview of expected future development steps of the Software FLO.



# Perishable inventory control with a service level constraint and non stationary demand

Alejandro G. Alcoba<sup>1</sup>, Eligius M.T. Hendrix<sup>1</sup>, Inmaculada García<sup>1</sup>, Gloria Ortega<sup>2</sup>, Karin G.J. Pauls-Worm<sup>3</sup>, and Rene Haijema<sup>3</sup>

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**Abstract.** The determination of order quantities in an inventory control problem of perishable products with non-stationary demand can be formulated as a Mixed Integer Nonlinear Programming problem. One of the challenges is to deal with the service level constraint in terms of the loss function. The properties of the optimal solution are studied and an specific algorithms to determine (near) optimal quantities is derived.

## 1 Introduction

The basis of our study is a Stochastic Programming (SP) model for a practical production planning problem over a finite horizon of  $T$  periods of a perishable product with a fixed shelf life of  $J$  periods. Items of age  $J$  cannot be used at the next period and are considered waste. The stochastic demand implies that the model has random inventory variables  $\mathbf{I}_{jt}$ . To keep waste due to out-dating low, one issues the oldest product first, i.e. FIFO issuance. The model we investigate aims to guarantee a service level constraint; the supplier guarantees to supply at least a fixed percentage of the expected demand for every period. The demand that cannot be fulfilled is not backlogged.

The model has to keep track of the lost sales  $\mathbf{X}_t$  in periods where demand exceeds the available amount of product. The service level constraint for every period is

$$E(\mathbf{X}_t) \leq (1 - \beta)\mu_t, \quad t = 1, \dots, T. \quad (1)$$

where  $\beta$  represents the service level required. This constraints are non linear and represents the so called *Loss Function*. Given a random variable  $\omega$  of demand, the first order loss function is defined as  $\mathcal{L}_\omega(x) = E_\omega[\max(\omega - x, 0)]$ . This function does not accept, in general, a closed formulation.

When considering a cost function including the costs of ordering and the costs of the amount produced, stored and wasted, an algorithm proposed can determine (near) optimal quantities.

**13<sup>th</sup> EUROPT Workshop on Advances in Continuous  
Optimization (EUROPT2015)**

**July 8-10, 2015, Edinburgh**

**Nonlinear scalarization mappings for set  
orderings\***

C. Gutiérrez<sup>†</sup>    B. Jiménez<sup>‡</sup>    E. Miglierina<sup>§</sup>    E. Molho<sup>¶</sup>

**Abstract**

This talk focuses on scalarization mappings for sets of a topological linear space and the so-called lower set less order relation. It is motivated by solution concepts of a set-valued optimization problem based on the set approach. To be precise, three scalarization schemes that generalize the Gerstewitz nonconvex separation functional and the oriented distance are studied, taking special attention to their monotonicity and order representation properties. Moreover, relationships between them are emphasized.

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# Infinite or finite-dimensional complementarity reformulation for the departure-time choice equilibrium problem with discrete multiple bottlenecks

Takashi Akamatsu (Tohoku University)

Kentaro Wada (The University of Tokyo)

Shunsuke Hayashi (Tohoku University)\* speaker

## Abstract

In this study, we provide a transparent approach to the analysis of dynamic user equilibrium and clarifies the properties of a departure-time choice equilibrium of a corridor problem involving discrete multiple bottlenecks. We reformulate the equilibrium problem as the linear/nonlinear complementarity problems by using the Lagrangian-like coordinate system instead of the existing Eulerian coordinate system. Then we analyze the existence and uniqueness of the equilibria. We also report some numerical observations.

# Simplicial branch and bound based on the upper fitting, longest edge bisection <sup>★</sup>

E.M.T. Hendrix<sup>1</sup>, J.F.R. Herrera<sup>2</sup>, L.G. Casado<sup>2</sup>, and I. García<sup>1</sup>

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**Abstract.** Simplicial partitions to divide a bounded area in branch and bound makes the use of an upper fitting appropriate for finding the bounds on the subsets. Bisecting the longest edge avoiding needle-shaped simplices leads to a choice of which longest edge to bisect in higher dimensions. We investigate the behaviour of the search and the resulting binary search tree depending on the selection rule for the longest edge.

**Keywords:** branching rule, upper fitting, simplices.

## 1 Problem formulation

We focus on the multidimensional box-constrained GO problem. The goal is to find at least one global minimum point  $x^*$  of

$$f^* = f(x^*) = \min_{x \in X} f(x), \quad (1)$$

where the feasible area  $X \subset \mathbb{R}^n$  is a nonempty box-constrained area. Given a global minimum point  $x^*$ , let scalar  $K$  be such that

$$K \geq \max_{x \in X} \frac{|f(x) - f^*|}{\|x - x^*\|}. \quad (2)$$

The function  $f^* + K\|x - x^*\|$  is an upper fitting according to [1] for an arbitrary  $x \in X$ . Consider a set of evaluated points  $x_i \in X$  with function values  $f_i = f(x_i)$ , then the area below  $\varphi(x) = \max_i \{f_i - K\|x - x_i\|\}$  cannot contain the global minimum  $(x^*, f^*)$ . Let  $f^U = \min_i f_i$  be the best function value of all evaluated points, i.e., an upper bound of  $f^*$ . Then the area  $\{x \in X : \varphi(x) > f^U\}$  cannot contain the global minimum point  $x^*$ .

We investigate the resulting binary tree for several selection rules for the longest edge in simplicial branch and bound. Notice that only in dimensions higher than three, there is a choice.

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## EUROPT 2015 – Abstract

Title: **FAIPA\_GSDP - A Feasible Arc Interior Point Algorithm for General Nonlinear SDP**

**José Herskovits<sup>1</sup>, Jean R. Roche<sup>2</sup>, Elmer G. Bazán<sup>3</sup> and Andrés Zuñiga<sup>4</sup>**

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**Key Words:** *General Semidefinite Programming, Nonlinear Smooth Optimization*

We consider optimization problems with an objective function subject to equality and inequality constraints as well as semidefinite constraints on symmetric matrix-valued functions. The involved functions are not necessarily linear or convex.

The present algorithm is an extension of FAIPA for nonlinear constrained optimization. FAIPA\_SDP makes iteration in the primal and dual variables to solve first order optimality conditions of the problem. Given an initial interior point, FAIPA generates a descent interior sequence, converging to a local solution of the problem.

At each iteration, FAIPA\_SDP defines a feasible descent arc and makes a line search along this arc, looking for a new interior point with lower objective. It requires the solution of three systems of linear equations with the same matrix. The first one generates a descent direction of the cost function. In the second linear system, a precisely defined perturbation in the left hand side is done and, as a consequence, a descent feasible direction is obtained. The third system computes an estimate of constraint's curvature in order to get a feasible descent arc. An inexact line search along this arc is then performed, to ensure that the new iterate is interior and the objective is lower.

We describe two formulations; one of them takes advantage of the matrices symmetry while the other maintains unchanged the structure of the constraint matrices. This is relevant in the case of applications where the SDP matrices has a favorable structure.

Some models for structural optimization involving SDP are presented and a set of test problems is solved. The problems were solved very efficiently without need of tuning parameters, suggesting robustness of the present approach.

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# Continuous Optimal Control Approaches to Microgrid Energy Management

Benjamin Heymann, J. Frédéric Bonnans, Pierre Martinon,  
Francisco Silva, Fernando Lanas and Guillermo Jimenez

## Abstract

We propose a novel method for the microgrid energy management problem by introducing a continuous-time, rolling horizon formulation. The energy management problem is formulated as a deterministic optimal control problem (OCP). We solve (OCP) with two classical approaches: the direct method, and Bellman's Dynamic Programming Principle (DPP). In both cases we use the optimal control toolbox BOCOP for the numerical simulations. For the DPP approach we implement a semi-Lagrangian scheme adapted to handle the optimization of switching times for the on/off modes of the diesel generator. The DPP approach allows for an accurate modeling and is computationally cheap. It finds the global optimum in less than 3 seconds, a cpu time similar to Mixed Integer Linear Programming (MILP) approaches. We achieve this performance by introducing a trick based on the Pontryagin Maximum Principle (PMP). The trick increases the computation speed by several orders and also improves the precision of the solution. The simulations use datasets from an actual microgrid located in northern Chile.

**Marcus Hillmann,**  
**Institute for Mathematics, Martin Luther University Halle-Wittenberg**

**Application of nonconvex subdifferentials for solving semi-obnoxious facility location problems**

The problem of locating a new facility with simultaneous consideration of existing attraction and repulsion points is a single-objective nonconvex location problem with great practical relevance. We will present a new approach for (approximately) solving such problems using nonconvex subdifferentials. While there are many theoretical results on these subdifferentials, it is rarely possible to explicitly calculate them. We will show that by taking advantage of the special structure of the mentioned problems, it is here for once possible to precisely calculate the corresponding subdifferentials. Furthermore, we will use these results to establish algorithms for solving the semi-obnoxious facility location problem based on different kinds of distance functions. At the end of the talk, we will give an outlook on possible future developments.

## **A Heuristic Algorithm to Solve Toll Optimization Problems**

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During the early years of industrial development, the production facilities were established near the consumers because the transportation was expensive, time-consuming, and risky. When transportation systems appeared, they allowed the producer to compete in distant markets, promoting economies of scale by increasing sales volume.

Due to the complexity of products and globalization, supply and distribution chains have grown enormously, therefore, logistics costs have “rocketed up” sharply. According to the data from the IMF (International Monetary Fund), logistics costs represent 12% of gross national product, and they range from 4% to 30% of the sales at the enterprise level.

Because of this growth, many countries have attached great importance to the development and modernization of the infrastructure to achieve greater participation in the global economy. There are organizations that deal with the development of communications and transportation infrastructure, building technologies to increase the coverage, quality and competitiveness of the infrastructure. In North America, administration of new (private) roads is commonly conceded to private companies, state governments, or financial institutions (banks, holdings, etc.), who set transportation tolls in order to retrieve money from the drivers.

It has been recently noticed that under the concession model, there is less traffic flow using these tolled highways. One of the strategies taken to increase the use of toll roads is the regulation of tolls (pass rates). However, what are the appropriate criteria to assign these?

Hence, this is the problem of assigning optimal tolls to the arcs of a multi-commodity transportation network. The toll optimization problem (TOP) can be formulated as a bilevel mathematical program where the upper level is managed by a firm (private or public) that raises revenues from tolls set on (some) arcs of the network, and the lower level is represented by a group of users traveling along the cheapest paths with respect to a generalized travel cost. The problem can be interpreted as finding a trade-off among tolls generating high revenues and those attractive for the users.

The aim of the present work is to propose an algorithm based on the allowable ranges to stay optimal (ARSO) resulting from sensitivity analysis after solving the lower level problem. With this powerful tool, one can analyze possible changes in the coefficients of some variables in the objective function within these allowed ranges that do not affect the optimal solution.

In addition to dealing with the allowable ranges, the proposed technique also uses the concept of “filled function”, which is applied under the assumption that the local maximum (in our case) has been found. Then the “filled function” technique helps one either find another local maximum, better than the previous ones, or determine that we have found the best feasible optimal solution, according to certain parameters of tolerance.

The validity and reliability of this technique are illustrated by the results of numerical experiments with test examples used to compare the proposed approach with the existing ones. The reported numerical results also confirm the robustness of the present algorithm.



# **Nonconvex Vectorization Derived by an Extension of Gerstewitz's Function**

**Emrah Karaman, Mahide Küçük, Yalçın Küçük,**

**İlknur Atasever Güvenç, Mustafa Soyertem, Didem Tozkan**

Vectorization is a reduction technique that replaces a set valued optimization problem with a vector optimization problem. In this work, by using an extension of Gerstewitz's function, a vectorization function is defined to reduce a set valued optimization problem given with respect to set less ordering. Some properties of this function are studied. In addition, relationships between set valued optimization problems and vector optimization problems, derived by vectorization of this set valued optimization problem, are examined. Furthermore, necessary and sufficient optimality conditions are presented without any convexity assumption.

# Characterization of efficient solutions in nonconvex vector optimization

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This paper presents existence conditions and characterization theorems for minimal points of nonconvex vector optimization problems in reflexive Banach spaces. Characterization theorems use special class of monotonically increasing sublinear scalarizing functions which are defined by means of elements of augmented dual cones. It is shown that the Hartley cone-compactness is necessary and sufficient to guarantee the existence of a properly minimal point of the problem. The necessity is proven in the case of finite dimensional space.

**Keywords:** vector optimization, nonlinear separation theorem, sublinear scalarizing functions, conic scalarization method, proper efficiency, existence theorem

# **Subgradient based solution method in nonconvex optimization**

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This talk presents the modified subgradient algorithm for solving constrained optimization problems without convexity and differentiability conditions. This method is based on the sharp augmented Lagrangian duality scheme where the zero duality gap condition is given in terms of weak subgradients. To update the dual variables, the direction of a subgradient of the dual function is used. The presented work relaxes the condition on the use of global optimal solutions of the subproblem at every iteration and proves the convergence theorem. The performance of the method is demonstrated on test problems and computational results are reported.

# Pseudo-Smooth Functions and Newton-Type Methods for Nonlinear Optimization and Complementarity Problems

Diethard Klatte (Speaker) <sup>1</sup>

Bernd Kummer <sup>2</sup>

A *pseudo-smooth* function  $f : \mathbb{R}^n \rightarrow \mathbb{R}^m$  is a locally Lipschitz function being continuously differentiable on an open and dense subset  $\Omega \subset \mathbb{R}^n$ . An important special class is that of (locally)  $PC^1$  functions. The so-called *small B-subdifferential* is the set  $D^\circ f(x) = \text{Limsup}_{\Omega \ni \omega \rightarrow x} Df(\omega)$ . In this talk we show how the generalized derivative  $D^\circ f$  can be applied for (parametric) Newton-type methods and characterize, depending on appropriate reformulations of nonlinear optimization or variational problems by Lipschitz equations, the concrete form of the Newton-auxiliary problems.

This approach was introduced and studied by B. Kummer in his paper "Generalized Newton and NCP- Methods: Convergence, regularity, actions" in *Discussiones Mathematicae - Differential Inclusions, Control and Optimization 20 (2000) 209-244* and also extensively handled by the authors in their book *Nonsmooth Equations in Optimization (Kluwer 2002)*.

In the present talk we focus on applications to Karush-Kuhn-Tucker systems of nonlinear optimization problems and to complementarity problems. Though the reformulations arise by different approaches, e.g. by using so-called Kojima systems or NCP functions, it turns out that all auxiliary problems are SQP-models, which differ alone by the weights for the constraints and the terms of the objective.

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# **A Multi-Objective Programming Approach with Different Importance and Priorities for Optimum Investment Decisions**

Ozan Kocadağlı

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## **Abstract**

This study proposes a novel fuzzy portfolio selection model that takes into accounts the risk preferences in accordance with the market moving trends as well as the risk – return tradeoff, and allows the decision makers to define a certain importance and priority among their objectives. To construct this model, firstly the portfolio return, risk and beta coefficient are assumed as main objectives including the possibilistic uncertainties. To define possibilistic uncertainty, the specific fuzzy membership functions are constituted for these objectives with respect to the risk preferences of investors and market moving trends. By means of the fuzzy goal programming techniques, a novel portfolio selection model is developed using these specific fuzzy membership functions. In the application section, three investment terms are examined in the Istanbul Stock Exchange National 30 Index (ISE30). While the first two implementations include the scenarios of ISE30 index having upward (bullish) and the downward (bearish) moving trends, the third implementation deals with a scenario in which the investors desire to chase the ISE30 index. In the analyses, the proposed model is compared with the classical Mean-Variance, Mean-Absolute-Deviation and Maxmin models in terms of their portfolio returns based on the selling prices in the test periods.

**Keywords:** Multiple Objective Programming, Fuzzy Goal Programming, Portfolio Selection Model, Risk Preferences of Investors, Capital Asset Pricing Model.

## **A first-order multigrid method for convex optimization**

Michal Kocvara (speaker) and Sudaba Mohammed (both University of Birmingham)

The aim of this talk is to design an efficient multigrid method for constrained convex optimization problems arising from discretization of some underlying infinite dimensional problems. Due to problem dependency of this approach, we only consider bound constraints with (possibly) a linear equality constraint. As our aim is to target large-scale problems, we want to avoid computation of second derivatives of the objective function, thus excluding Newton like methods. We propose a smoothing operator that only uses first-order information and study the computational efficiency of the resulting method.

# Output analysis and stress testing for risk constrained portfolios

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**Abstract:** Solutions of portfolio selection problems are often influenced by the model misspecification and simplifications, or by errors due to approximations, estimations and incomplete information. The obtained optimal investment strategies, recommendations for the risk and portfolio manager, should be then carefully analyzed. We shall deal with output analysis, robustness and stress testing with respect to uncertainty or perturbations of input data for static risk constrained portfolio optimization problems via the contamination technique and the worst-case analysis. We shall focus on the mean-risk models and the second order stochastic dominance constrained problems under suitable smoothness and/or convexity assumptions that are fulfilled e.g. for the Markowitz mean-variance model. The presented detailed numerical illustrations concern stress testing for scenario-based mean-risk problems with the CVaR objective or CVaR constraints.

**Keywords:** stochastic programming, contamination, stress testing, mean-risk model, stochastic dominance

# Tropical Optimization Problems: Solution Methods and Application Examples

Nikolai Krivulin

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Optimization problems constitute a notable research and application area in tropical mathematics focused on the development of new solutions to known and emerging problems in operations research and other fields. Tropical mathematics, which is concerned with the theory and applications of semirings with idempotent addition, dates back to the pioneering studies of R. A. Cuninghame-Green, B. Giffler, A. J. Hoffman, S. N. N. Pandit, N. N. Vorob'ev, and I. V. Romanovskiĭ published in early 1960s.

In the next decades, optimization problems that can be formulated and solved in the framework of tropical mathematics were investigated in many publications, among which are monographs by R. A. Cuninghame-Green, U. Zimmermann, P. Butkovič, and a number of research papers. For some problems, complete direct solutions were obtained in a closed form under fairly general assumptions. Other problems are known to have only algorithmic solutions in the form of an iterative computational procedure to provide a particular solution, if any, or to show that no solution exists, otherwise.

We consider optimization problems in the general setting of tropical mathematics to minimize nonlinear functions that are defined on vectors over an idempotent semifield (semiring with multiplicative inverses) by using a multiplicative conjugate transposition operator. Both unconstrained problems and problems with constraints given by vector equations and inequalities are examined. To solve the problems, we use techniques based on the derivation of lower bounds on the objective function and on the solution of linear vector equations and inequalities. Under fairly general conditions, new direct solutions are given in a unified closed vector form. For many problems, the solutions obtained are complete solutions. The calculation of the solutions involves simple matrix-vector computations, which offers ease of implementation and provides low computational complexity. We discuss applications to real-world problems in various fields, including location analysis, activity scheduling, and decision making, and present examples.



# Application of the Sequential Quadratic Programming Method to Finding Error Trajectories of Hybrid Dynamical Systems

Jan Kuřátko\* and Stefan Ratschan†

March 27, 2015

A hybrid dynamical system is a dynamical system that features both continuous and discrete state and behaviour. For example, a thermostat controlling the temperature in a room can be modeled as a hybrid dynamical system: When the thermostat is switched on, the temperature rises, and when it is switched off, the temperature decreases. States *off* and *on* are discrete, and the temperature in the room is a continuous state of the system.

We are concerned with the problem of finding a trajectory of a given hybrid dynamical system that originates in a given set of initial states and reaches a given set of unsafe states. We call such a trajectory from an initial to an unsafe state an *error trajectory*. One may view this problem as a variation of the classical boundary value problem. However, unlike the classical boundary value problem, in this case we search for error trajectories of *arbitrary length*.

We reformulate our problem as a continuous optimization problem and use the *Sequential Quadratic Programming* (SQP) method for finding error trajectories. We consider several different variants of the formulation, compare their numerical behaviour on a set of problems, and investigate advantages and disadvantages of each formulation. We present numerical results, and discuss sparsity of the resulting Hessian.

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# Black-box global optimization: deterministic and metaheuristic approaches

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**Keywords.** Global optimization, black-box problems, deterministic methods, nature-inspired algorithms.

Global optimization problems are considered where the objective function is a continuous, non-differentiable, and multiextremal function satisfying the Lipschitz condition over a hyperinterval with an unknown Lipschitz constant. It is also supposed that the function is given by a black-box and its evaluation at a point is a time-consuming operation. Many algorithms for solving this problem have been discussed in literature [1–6]. Among them, deterministic global optimization methods form a well-developed group with many important applications [3–5]. One of their main practical advantages is the possibility to obtain guaranteed estimations of global solutions and to demonstrate (under certain analytical conditions) useful global convergence properties.

In this talk, some deterministic approaches [4, 5] developed by the authors to construct black-box global optimization methods are discussed and compared with several metaheuristic nature-inspired algorithms [1, 2]. Numerical comparison is performed on test classes and on some practical engineering problems with the usage of different criteria [4–6].

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# Distributed Methods for Constrained Nonconvex Optimization Problems

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Lampariello · Gesualdo Scutari

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**Abstract** In this work we propose a general algorithmic framework for non-convex constrained optimization problems. The aim of the scheme is to compute stationary solutions of the original problem by solving a sequence of (separable) strongly convex subproblems, while preserving feasibility of the iterates. The algorithm is amenable to parallel and distributed implementations. Moreover, we show the wide applicability of the scheme, which compares favorably to existing methods and presents features that are new in the literature. We illustrate the application of the method to several optimization problems in communications and networking.

# AN INEXACT PROXIMAL REGULARIZATION METHOD FOR UNCONSTRAINED OPTIMIZATION

Paul ARMAND and Isai LANKOANDE

We present an inexact proximal algorithm to solve a smooth unconstrained minimization problem. It is known that this kind of algorithm is suitable to solve a degenerate problem, when the Hessian is singular at a local optimal solution. The main feature of our algorithm is that it uses an outer/inner iteration scheme, in which an outer iteration is performed after the update of the regularization parameter, and is possibly followed by a sequence of minimization steps of the proximal function. We show that the algorithm has a strong global convergence property under mild assumptions. We also present an asymptotic analysis of the algorithm under the local error bound condition. Numerical experiments are reported.

## An Improved Full Nesterov-Todd Interior-Point Algorithm for Symmetric Optimization

Dr. Goran Lesaja, Department of Mathematical Sciences, Georgia Southern University, USA

**Abstract:** In this talk, an improved complexity analysis of full Nesterov-Todd step feasible interior-point method for symmetric optimization is considered. Using several new results from Euclidean Jordan algebras and associated symmetric cones, a sharper quadratic convergence result than previously known is established, leading to a wider quadratic convergence neighborhood of the central path for the iterates in the algorithm. However, the best known iteration bound for full Nesterov-Todd step feasible interior-point methods is still achieved.

## **Mixed-Integer PDE-Constrained Optimization**

Sven Leyffer

Many complex applications can be formulated as optimization problems constrained by partial differential equations (PDEs) with integer decision variables. Examples include the remediation of contaminated sites and the maximization of oil recovery; the design of next-generation solar cells; the layout design of wind-farms; the design and control of gas networks; disaster recovery; and topology optimization.

We will present emerging applications of mixed-integer PDE-constrained optimization, review existing approaches to solve these problems, and highlight their computational and mathematical challenges. We introduce a new set of benchmark set for this challenging class of problems, and present some early numerical experience using both mixed-integer nonlinear solvers and heuristic techniques.

# On polynomial sized representations of Hilbert's identity and moments tensors

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## Abstract

Hilbert's identity is an important tool for the analysis of Hilbert's 17th problem. The identity asserts that the polynomial function  $(x_1^2 + x_2^2 + \cdots + x_n^2)^d$  can be expressed as a sum of powered linear terms. However, in Hilbert's original construction, the number of powered linear terms is exponential in  $n$ . For  $d = 2$ , we present efficient algorithms to construct a sum of powered linear terms, where the number of terms is no more than  $2n^4 + n$ . Moreover, we prove that the number of required linear terms is at least  $n(n+1)/2$  for  $d = 2$ . Our tools in the construction rely on a new notion called  $k$ -wise uncorrelated random variables.

Our results immediately imply that it is possible to find a  $(2n^4 + n)$ -point distribution whose fourth moments tensor is exactly the symmetrization of  $Q \otimes Q$ , where  $Q$  is a positive semidefinite matrix. Extensions of the results to complex tensors are discussed as well. As an application, we answer an open question to assert that the computation of the matrix  $2 \mapsto 4$  norm is NP-hard in general. Finally we discuss a polynomial sized representation of Hilbert's identity for general degree  $d$ .

This is a joint work with Bo Jiang, Simai He and Shuzhong Zhang.

**Keywords:** moments tensors, Hilbert's identity, uncorrelated random variables, matrix norm.

# Unifying semidefinite and set-copositive relaxations of binary problems and randomization techniques

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## Abstract

A reformulation of quadratically constrained binary programs as duals of set-copositive linear optimization problems is derived using either  $\{0, 1\}$ -formulations or  $\{-1, 1\}$ -formulations. The latter representation allows an extension of the randomization technique by Goemans and Williamson. An application to the max-clique problem shows that the max-clique problem is equivalent to a linear program over the max-cut polytope with one additional linear constraint. This transformation allows the solution of a semidefinite relaxation of the max-clique problem with about the same computational effort as the semidefinite relaxation of the max-cut problem – independent of the number of edges in the underlying graph. A numerical comparison of this approach to the standard Lovasz number concludes the talk.



## Parallel Adaptive Preconditioners for Sequences of KKT Systems

**Daniel Loghin** (*School of Mathematics, University of Birmingham*)

In this work we consider the design of parallel adaptive solvers for sequences of large, sparse linear systems of equations in KKT form. Problems of this type are common in the context of solving nonlinear saddle-point systems, or in interior point methods. We describe an approach which involves (algebraic) domain decomposition methods coupled with adaptive interface preconditioners.

Domain decomposition methods are established techniques for solving linear systems arising from the discretization of PDE. A key feature of these methods is the solution of the interface problem (or interface Schur complement) arising from a non-overlapping decomposition of the domain. For KKT matrices, this interface matrix has a saddle-point structure, is often ill-conditioned and does not afford in general an obvious preconditioner. Our focus will be on re-using Krylov information generated at each step for the purpose of constructing an enhanced, adaptive interface preconditioner.

Numerical results using test problems arising in topology optimization and non-Newtonian flow modeling are included to illustrate the procedure and verify the optimality of the proposed solver technique with respect to parameters such as the problem size and number of subdomains.

## **Fülöp's equivalence between a linear bilevel programming problem and linear optimization over the efficient set revisited**

Andreas Löhne

In the seminal paper by Fülöp [1] it was shown that a linear bilevel programming problem is equivalent to the problem to minimize a linear function over the set of efficient (or non-dominated or minimal) points of a multiple objective linear program. We discuss Fülöp's results and some extensions. In particular, we replace the multiple objective program by a vector linear program in order to can reduce the number of objectives by one. We further discuss the role of vector linear programming duality.

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**A filter-based dynamically dimensioned search algorithm for constrained global optimization**

Maria Joseane F. G. Macêdo · M. Fernanda P. Costa · Ana Maria A. C. Rocha · Elisabeth Wegner Karas

**Abstract**

This work presents a filter-based dynamically dimensioned search (FDDS) algorithm for solving nonconvex constrained global optimization problems. The FDDS is an extension of the dynamically dimensioned search algorithm, which is a stochastic heuristic method based on dynamic adjustment of a subset of coordinates of the best solution. The proposed method uses a filter technique to handle the constraints, reformulating the optimization problem as a bi-objective one. Computational experiments are carried out with a set of benchmark global test problems available in the literature to analyze the performance of FDDS algorithm. The results are compared with other stochastic methods and evaluated in terms of quality of solutions and computational effort required.

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# Operator splitting techniques for a stochastic multi-zonal long-term energy production planning problem

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**Mots-clefs :** stochastic optimization, decomposition methods

We propose here to investigate new variants of proximal splitting methods (like the family of Douglas-Rachford splitting) and test them on a complex stochastic optimization problem derived from a real-life long-term energy planning model.

We consider a set of geographical zones  $Z$  interconnected by a network of economic agreements to import or export electricity. Each arc  $e = (z, z')$  carries the flow of imported energy denoted by  $f_{et}$  in each period of the time horizon  $t \in [1, \dots, T]$ . We denote  $d_{zt}$  the total demand and  $i_{zt}$  the input of water resource for zone  $z \in Z$  at step  $t$  (random information). The variables are the local production levels  $p_{zt}$ , the control of hydroelectrical reserve  $u_{zt}$  and the interzonal flows  $f_{et}$ . The multistage stochastic program we face up is the following :

$$\begin{aligned} \min_{p, x, u \in L^2} \quad & \mathbb{E} \left[ \sum_{t=0}^{T-1} (\sum_{z \in Z} c_z(p_{zt}) + \sum_{e \in E} l_e(f_{et})) \right] \\ \text{s.t} \quad & p_t + u_t - A f_t = d_t \end{aligned} \tag{1}$$

$$x_{z,t+1} = x_{zt} - u_{zt} + i_{zt} \quad \forall z \in Z, \forall t \tag{2}$$

$$+ \text{Zonal constraints on random variables } p_{zt}, u_{zt}, f_{et} \tag{3}$$

We apply the Douglas-Rachford (or equ. Proximal Decomposition, see [1]) splitting method to the dynamic reformulated model. It consists in solving by a Dual Dynamic Programming technique the local subproblems :

$$\min_{(q_z, u_z, f_z, \phi_z)} C_z(q_z, u_z, f_z, \phi_z) + 1/2\lambda(\|f_z - f_z^k - \lambda v_z^k\|^2 + \|\phi_z - \phi_z^k + \lambda w_z'^k\|^2)$$

update the dual variables and project back on a customized coupling subspace and its orthogonal (see details in [2] for the deterministic model).

Numerical results are presented on realistic instances with  $|Z| = 12, T = 365$  and piecewise linear convex production costs that show the efficiency of the decomposition approach.

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# INEXACT RESTORATION METHOD TO SOLVE THE DEMAND ADJUSTMENT PROBLEM

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The problem of estimating the origin-destination matrix (DAP: demand adjustment problem) in a congested transport network, can be dealt as a Mathematical Program with Equilibrium Constraints (MPEC), where the equilibrium constraint is precisely, the user's deterministic equilibrium formulated by Wardrop (DUE: deterministic user equilibrium). Under certain assumptions over the net it can be proved that equilibrium solutions coincide with the solutions of a convex optimization problem (TAP: traffic assignment problem). Consequently, DAP can be rewritten as a bilevel optimization problem.

An inexact restoration method for nonlinear bilevel problems is studied. The objective is to adapt it in order to test its performance over DAP. In its original version and under certain hypothesis convergence of the method is proved in [1].

So far, we have studied and proposed in [4], heuristics that deal with the reformulation of DAP as a single level problem. In contrast, the proposal of this work is interesting as it takes in to account and profits from the structure of the problem of the lower level, the TAP. The original motivation for studying this method is associated with this last observation, as there exist plenty of algorithms with very acceptable performance that solve TAP and that can be used in an implementation of the inexact restoration method to solve DAP. More precisely, the algorithms implemented in [2] will be used to solve the assignments in the restoration phase and for the optimization phase a minimization procedure inspired in the work of Martinez [3] will be used.

Numerical results will be presented.

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# Decomposition Techniques for Multi Layer Perceptrons Training

Luigi Grippo, Andrea Manno and Marco Sciandrone

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## Abstract

In this work we consider the learning problem of Multi Layer Perceptrons (MLPs) formulated as the problem of minimizing a smooth error function. As well-known the learning problem of MLPs can be a difficult nonlinear nonconvex optimization problem. Typical difficulties can be the presence of extensive flat regions and steep sided valleys in the error surface, and the possible large number of training data and of free network parameters. We define a wide class of batch learning algorithms for MLP, based on the use of block decomposition techniques in the minimization of the error function. The learning problem is decomposed into a sequence of smaller and structured minimization problems in order to advantageously exploit the structure of the objective function. Theoretical convergence results are established and a specific algorithm is constructed and evaluated through an extensive numerical experimentation. The comparisons with state-of-the-art learning algorithms show the effectiveness of the proposed techniques.

# A Projection Multi-objective SVM Method for Multi-class Classification

Ling Liu<sup>\*</sup>      Belén Martín-Barragán<sup>†</sup>      Francisco J. Prieto<sup>‡</sup>  
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## Abstract

Support Vector Machines (SVMs) have become a very popular technique in the machine learning field for classification problems. They were originally proposed for classifications of two classes. For multi-class classifications, various single-objective models have been proposed mostly based on two families of methods: an all-together approach and a combination of binary classifiers. However, most of these single-objective models consider neither the different costs of different misclassifications nor the user's preferences. To overcome these drawbacks, multi-objective models have been proposed. By solving a large second-order cone program (SOCP), these multi-objective models give us weakly Pareto-optimal solutions. The need of solving large SOCPs makes these multi-objective models impractical when we have a large amount of classes to classify. In this paper, we propose a multi-objective technique that we denominate Projected Multi-objective SVM (**PM**), which works in a higher dimensional space than the object space. For **PM**, we can characterize these Pareto-optimal solutions. And for classifications with large numbers of classes, with **PM**, we can significantly alleviate the computational bottleneck. From our experimental results, and compared with the single-objective multi-class SVMs (based on an all-together method, one-against-all method and one-against-one method), **PM** obtains comparable values for the training classification accuracies, testing classification accuracies and training time. Compared to other published multi-objective multi-class SVMs, **PM** gives higher Pareto-compliant indicator values while requiring less computation time.

**KEYWORDS:** Multi-class SVM; Multi-class multi-objective SVM; Weakly Pareto-optimal solution; Pareto-optimal solution

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# Solving MINLP with Heat Exchangers: Special Structure Detection and Large-Scale Global Optimisation

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Optimising heat exchangers networks (HEN) may increase efficiency in industrial plants; we develop deterministic global optimisation algorithms for a mixed-integer nonlinear optimisation (MINLP) model that simultaneously incorporates utility cost, equipment area, and hot / cold stream matches [1, 3]. In this work, we automatically recognise and exploit special mathematical structures common in HEN including log mean temperature difference and Chen approximation; we computationally demonstrate the impact on the global optimisation solver ANTIGONE [2] and benchmark large-scale test cases against heuristic approaches.

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# On the directional derivative of optimal value functions of nonsmooth convex problems

Robert Mohr<sup>1</sup> and Oliver Stein<sup>2</sup>

We present a formula for the directional derivative of the optimal value function of a nonsmooth and completely convex parametric problem. The formula is valid at boundary points of the domain of the optimal value function if the direction belongs to a certain conic set. We derive a functional description for this conic set and apply the formula to selected convex problems such as convex semi-infinite problems or problems involving sums and maxima of norms.

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# $H_\infty$ control synthesis under structural constraints based on Global Optimization

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Controlling an autonomous vehicle or a robot requires the synthesis of control laws for steering and guiding. To generate efficient control laws, a lot of specifications, constraints and requirements have been translated into norm constraints and then into an constraint feasibility problem. This problem has been solved, sometimes with relaxations, using numerical methods based on LMI (Linear Matrix Inequalities) or SDP (Semi Definite Program). The main limitation of these approaches are the complexity of the controller for implementation in an embedded system. But, if a physical structure is imposed to the law control in order to make easier the implementation, the synthesis of this robust control law is much more complex. And this complexity has been identified as a key issue for several years. A efficient first approach was given by Apkarian and Noll based on local non-smooth optimization.

In this talk, we will present a new approach based on **global optimization** in order to generate **robust control laws**. This new global optimization algorithm is based on interval arithmetic and contractor programming. Contractor Programming is a methodology which allows to enclose each algorithm in a unify framework, in order to interact heterogeneous formulations or techniques. It is a set-membership method, considering sets in place of floating points. Using Contractor Programming, we will show a user-friendly way to solve problems with non-smooth functions and constraints with quantifiers. The main idea consists in constructing a contractor based on the feasible region of the optimization problem and its complementary. This contractor is then used to accelerate the convergence of a Branch and Bound algorithm.

We will illustrate this new approach on a example on the control of a periodic second order system  $G$  with a PID controller  $K$  subject to two frequency constraints on the error  $e$  and on the command  $u$  of the closed loop system. The objective is to find  $k = (k_p, k_i, k_d)$  minimizing the  $H_\infty$  norm of the controlled system.

$$G(s) = \frac{k\omega_n}{s^2 + 2\xi\omega_n s + \omega_n^2}, \quad K(s) = k_p + \frac{k_i}{s} + k_d s.$$

The feasible region  $\mathbb{K}_{in}$  of our global optimization problem have the following form, with  $Re_1$  and  $Im_1$  the real and imaginary part of the transfer function  $C_1$  corresponding to the first constraint on the error, and  $Re_2$  and  $Im_2$  of  $C_2$  corresponding to the second constraint on the command. The objective function consists to minimizing  $\gamma$ .

$$\begin{aligned} \mathbb{K}_1 &= \{(k, \gamma) : \|C_1(G \star K)\|_\infty \leq \gamma\} = \{(k, \gamma) : \forall \omega, \sqrt{Re_1^2(k, \omega) + Im_1^2(k, \omega)} \leq \gamma\}, \\ \mathbb{K}_2 &= \{(k, \gamma) : \|C_2(G \star K)\|_\infty \leq \gamma\} = \{(k, \gamma) : \forall \omega, \sqrt{Re_2^2(k, \omega) + Im_2^2(k, \omega)} \leq \gamma\}, \\ \mathbb{K}_{in} &= \mathbb{K}_1 \cap \mathbb{K}_2. \end{aligned}$$

This algorithm is implemented in the library IBEX (<http://www.ibex-lib.org>) which is free available.

## **Risk-utility trade-off for a new method of statistical disclosure limitation based on a mixture model with constraints**

Anna Oganian

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National Center for Health Statistics (MD, US)

Before releasing databases which contain sensitive information about individuals, data publishers must apply Statistical Disclosure Limitation (SDL) methods to them, in order to avoid disclosure of sensitive information on any identifiable data subject. Transforming unsafe data into safe data can be viewed as an optimization problem whose solution is far from trivial. SDL methods aim to reduce the risk of disclosure of confidential information and at the same time to maximize the utility of the released data. These are conflicting goals.

In this work we describe a new SDL approach, which controls attribute disclosure risk for continuous variables. The proposed method is based on synthetic data generation using mixture model with constraints on parameters of components' spread. The method guarantees that the requirements of an attribute disclosure risk metric called  $v$ -dispersion are satisfied.

Experiments with real data show that our method compares very favorably with other methods of disclosure limitation for continuous microdata in terms of utility and risk.

**Keywords and phrases:** Statistical disclosure limitation (SDL),  $v$ -dispersion, mixture models, expectation-maximization (EM) algorithm, constraints.

# A variational inequalities approach for a closed-loop supply chain network under environmental regulations

E. ALLEVI<sup>1</sup>, A. GNUDI<sup>2</sup>, I.V. KONNOV<sup>3</sup>, AND G. OGGIONI<sup>4</sup>

## Abstract

Global climate change has encouraged international and regional adoption of pollution taxes and carbon emission reduction policies. Europe has taken the leadership in environmental regulations by introducing the European Union Emissions Trading System (EU-ETS) in 2005 and by developing and promoting a set of policies destined to lower carbon emissions from transport sectors. These environmental policies have significantly affected the production choices of European energy and industrial sectors.

In this paper, we propose a closed-loop supply chain network design problem that includes raw material suppliers, manufacturers, consumers, and recovery centers. The objective of this paper is to formulate and optimize the equilibrium state of this closed-loop supply chain network assuming that manufacturers are subject to the EU-ETS and a carbon tax is imposed on truck transport. The model is optimized and solved by using the theory of variational inequalities.

**Keywords:** Closed-loop supply chain network, environmental regulations, variational inequalities.

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## **Value Iteration versus Policy Iteration on Markov and Semi Markov Decision problem**

Adewoye, S. Olabode

Department of Mathematics, Yaba College of Technology, Nigeria.

The most interesting problem in the treatment of dynamic probabilistic system models arise when we have the opportunity to control the system. In such problems we want to select from among the alternatives available for the operation of the system that are most rewarding. In this paper, commonly used algorithms such as value iteration and several version of modified policy iteration are compared. Policy iteration is usually recommended as yielding better result. But our results show policy iteration applied to non-discounted model without special restrictions might not converge to an optimal policy. We proposed the combined value- policy iteration as a better option.

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**TITLE:**

Improving the splitting preconditioner for linear systems from interior point methods

**ABSTRACT:**

We are concerned with the linear systems arising when an interior point method is applied to solve large-scale linear programming problems. The choice of an effective preconditioner is essential for the success of the iterative methods approach for solving these systems. We propose a new ordering for the splitting preconditioner, taking advantage of the sparse structure of the original matrix. A formal demonstration shows that performing this new ordering, the condition number of the preconditioned matrix is limited. Case studies show that the proposed idea is competitive with direct methods because the linear system has a better condition number than the original one and with the new ordering, the running time is reduced.

## Two regularized primal-dual algorithms for nonlinear programming

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**Keywords :** Nonlinear programming, Primal-dual algorithms, Regularization

We present two regularized primal-dual algorithms for nonlinear programming. These algorithms are based on Newton-like methods applied to a perturbation of the optimality system that follows naturally from a reformulation of the initial problem using either a quadratic penalty function or an augmented Lagrangian function to handle equality constraints. The globalization is performed by applying a backtracking line search algorithm based on a primal-dual merit function. We detail the update rules of the different parameters in order to obtain good global convergence properties and a high rate of convergence. We also show that an advantage of this approach is to introduce a natural regularization of the linear system solved at each iteration to compute a search direction. This allows our algorithms to perform well when solving degenerate problems for which the Jacobian of constraints is rank deficient. Another important feature of these algorithms is that the penalty parameter is allowed to vary during the inner iterations, while it is usually kept constant. These methods have been implemented in the SPDOPT code. The good practical performances of this new solver have been demonstrated by comparing it to the reference codes IPOPT, ALGENCAN and LANCELOT.

# Precipitation Modeling by Polyhedral RCMARS and Comparison with MARS and CMARS

Ayşe Özmen<sup>1</sup>, Gerhard-Wilhelm Weber<sup>1</sup>, İnci Batmaz<sup>1,2</sup>

Nowadays, climate change is an important issue due to the possibility that it may result in extreme weather events such as floods or droughts. Therefore, precipitation forecasting has drawn attention since it is a useful tool in meeting the increasing necessity for the efficient management of water resources as well as in preventing disasters before they happen. Although, precipitation is a very complicated physical process in nature, which makes it difficult to forecast, recent positive developments in predictive data mining techniques used in early warning systems improve the accuracy of precipitation forecasts. In literature, there exist many statistical and computational methods used for this aim, including linear and nonlinear regression, kriging, time series models, neural networks and Multivariate Adaptive Regression Splines (MARS). Among these methods, MARS stands out as the better performing precipitation modeling method. In fact, MARS is very useful nonparametric technique to construct high-dimensional and nonlinear multivariate functions in many areas of engineering and science. In this study, we applied a recently developed method called Robust Conic MARS (RCMARS) to forecast precipitation. Here, in CMARS, which is developed as an alternative to MARS, the model complexity is penalized in the form of Tikhonov regularization and studied as a conic quadratic programming. In RCMARS, CMARS is refined further by including the existence of uncertainty in the future scenarios and robustifying it with a robust optimization technique. RCMARS is more model-based and employs continuous, well-structured convex optimization that enables us to use interior point methods and their codes, e.g., MOSEK. To evaluate the performance of the RCMARS method, it is applied to a precipitation model for the continental central Anatolia region of Turkey, where drought has been a recurrent phenomenon for the last few decades. Then, the performance of the RCMARS precipitation model is compared to that of MARS and CMARS. The results indicated that RCMARS constructs more precise and stable precipitation model compared to those of MARS and CMARS.

**Keywords:** Robust Conic Multivariate Adaptive Regression Splines, Robust Optimization, Conic Quadratic Programming, Continental Central Anatolia, Meteorological data, Early Warning System

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# Robust Multi-source multi-commodity capacitated Facility Location Problem (cFLP)

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## Abstract

Given a set of potential facilities  $F$  (with individual opening cost  $f_j$  and finite capacity  $s_j$ ), a set of clients  $C$  with variable demands, and a distance metric  $d(i, j)$  defined between client  $i$  and facility  $j$ , the capacitated Facility Location Problem (cFLP) consists in selecting a set  $S$  of facilities to open and an assignment of demands to open facilities that minimize the sum of facility opening cost, supplying cost and connection cost (total distance between every client and the facility it is connected to). This problem shares many similarities with the one consisting in minimizing the cost for opening a set of servers or replicas for storing a set of digital objects (data) and connecting clients to facility locations so as to satisfy their demands. The multi-source dimension of the problem translates the situation where the same data object may be available simultaneously at different facility locations. When data objects of different types are available simultaneously at multiple facility locations, the problem shares the characteristics of a multi-product or multi-commodity model. Moreover the dynamics of client demands may lead to consider the replication of data objects at multiple facility locations depending on the capacity allocation and sharing model together with their associated constraints; therefore, client demands can be possibly assigned to locations other than the closest facility. Nevertheless, some specific characteristics of our problem need to be further emphasized as data objects are not physical goods in contrast to classical facility location problems, i.e., servicing one customer doesn't prevent servicing another customer requesting for the same data object.

On the other hand, due to the inherent variability of client demands resulting from many exogeneous dependencies, the input data to the problem is subject to uncertainty. To take this uncertainty into account as part of the problem formulation, we customize robust optimization methods to find solutions which remain valid even if the input data (the spatio-temporal properties of client demands) changes. We further consider the set induced robust optimization method where the uncertain data are assumed to be varying in a given uncertainty set  $\mathcal{U}$ . The goal is to choose the best solution among those protecting against data uncertainty, i.e., among the candidate solutions that remain feasible for all realizations of the scenarios described by the parameters appearing in the constraints and/or the objective function and belonging to the uncertainty set  $\mathcal{U}$ . Optimizing the worst-case over all scenarios, referred to as strict robustness contrasts to less conservative methods which overcome usual situations (appearing in practical applications) where the set of strictly robust solutions is actually empty, or all of the strictly robust solutions lead to undesirable solutions (i.e., when yielding inadequate objective function values). Among these less conservative methods, the  $\Gamma$ -robustness method (and its variants) which relies on the working assumption that it is unlikely that all coefficients of one constraint change simultaneously to their worst-case values, limits the uncertainty set  $\mathcal{U}$  by fixing a real number for every constraint and protect against the case where that at most  $\Gamma$  coefficients deviate. For this purpose, we further assume that i) the client demands are uncertain for all the commodities, i.e., their value is not known exactly when the optimization problem is solved and ii) data uncertainty can be modeled by means of box+polyhedral uncertainty sets leading to consider client demands within their estimated value and its distributional deviation around that value. On the other hand, dealing with uncertainty in client demands may be considered in the context of both a single-period formulation (where demands may vary within a single period due to the dependence on the requested object) and a multi-period formulation (where demands dynamically vary over time due to their dependence on the client utility, activity, and expectation).

In this paper, we propose and compare different robust formulations (depending on the features of the uncertainty sets) together with resolution methods of the multi-source multi-commodity capacitated facility location problem adapted for replicas placement and data replication. Results of numerical experiments performed on representative settings and running scenarios are reported and analyzed. We further investigate the dependence of the solution properties and the computational time to produce them with respect to the size of the instances under consideration. Finally, we exploit these results to compare different replicas placement and data replication strategies against various capacity allocation and resource sharing models between replicas.

## **Computational Models and Challenging Optimization Problems**

Panos M. Pardalos

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Most of the conventional computer models are based on the von Neumann computer architecture and the Turing machine model. However, quantum computers (several versions!), analog computers, dna computers, and several other exotic models have been proposed in an attempt to deal with intractable problems. We are going to give a brief overview of different computing models and discuss several classes of optimization problems that remain very difficult to solve. Such problems include graph problems, nonlinear assignment problems, and global optimization problems. We will start with a historical development and then we will address several complexity and computational issues. Then we are going to discuss heuristics and techniques for their evaluation.

## **A Multilevel Proximal Algorithm for Large Scale Composite Convex Optimization**

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Abstract: Composite convex optimization models consist of the minimization of the sum of a smooth convex function and a non-smooth convex function. Such models arise in many applications where, in addition to the composite nature of the objective function, a hierarchy of models is readily available. We adopt an optimization point of view and show how to take advantage of the availability of a hierarchy of models in a consistent manner. We do not use the low fidelity model just for the computation of promising starting points but also for the computation of search directions. We establish the convergence and convergence rate of the proposed algorithm and discuss numerical experiments.

# Cutting surface methods for equilibria

Giancarlo Bigi\*

Giandomenico Mastroeni\*

Mauro Passacantando\*

**Abstract.** The abstract equilibrium problem (EP) provides a rather general setting which includes several mathematical models such as optimization, variational inequalities, fixed point and complementarity problems, Nash equilibria in noncooperative games. It is well known that a pseudomonotone EP is equivalent to minimize the so-called Minty gap function. Though it is a convex function, it can be difficult to evaluate since this requires to solve nonconvex optimization problems. The aim of this talk is to present cutting type methods for solving EP via the Minty gap function, relying on lower convex approximations which are easier to compute. These methods actually amount to solving a sequence of convex optimization problem, whose feasible region is refined by nonlinear convex cuts at each iteration. Convergence is proved under suitable monotonicity or concavity assumptions. The results of preliminary numerical tests on Nash equilibrium problems with quadratic payoffs, other linear EPs and variational inequalities are also reported.

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## **Hedging the Risk of Renewable Energy Sources in Electricity Production**

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Renewable Energy Sources (RES) have become a significant part of electricity production in Europe due to the 20-20-20 targets. The low operating costs of RES plants with respect to conventional plants represent a positive feature of RES, but, unfortunately, come with an increased intermittence of energy supplied to the grid. This trade-off between costs and volatility of energy production has required the introduction of more sophisticated risk management tools by energy producers.

In this paper, we propose an equilibrium model of the electricity market, which allows for capacity expansion of both conventional and wind plants, and investigate the effects of wind energy penetration. The agents are assumed to be either risk-neutral or risk-averse. Besides, they hedge the intermittence of wind by buying option contracts written on a suitable index.

The model is formulated as a complementarity problem and is implemented in GAMS, while the option contracts are priced through Monte Carlo methods. Intermittence of wind is accounted for through scenario generation.

\*Speaker

# An optimization based approach to large scale flow simulation in fractured media

Sandra Pieraccini\*, joint work with S. Berrone\* and S. Scialò\*

The simulation of flows in fractured media is a challenging issue relevant in several critical applications (Oil&Gas enhanced production, geothermal applications...). We consider in this context the Discrete Fracture Network (DFN) models. DFNs are given by a (possibly large) number of planar polygons in the 3D space, stochastically distributed, resembling the fractures in the underground. The quantity of interest is the flow potential, called hydraulic head. The flow on each fracture is ruled by the Darcy law, and flux exchange among fractures occurs through fracture intersections. Suitable matching conditions are therefore imposed at fracture intersections in order to ensure continuity of the hydraulic head and flux balance. These matching conditions strongly couple the local equations defined on the fractures. Common issues to be tackled are the complexity of the domain and the huge computational cost. Indeed, very large scale problems are encountered, when performing simulations at basin scale, since the number of fractures involved may realistically count even  $10^6$  fractures. Furthermore, one of the major complexities related to standard approaches is the construction of good quality computing grids on the fractures for the underlying space discretization. Indeed, if some mesh conformity is required along traces intersection, the meshing process may result in an excessively fine, poor quality mesh, or it may even result infeasible.

In recent work, we proposed a novel approach for flow simulations on arbitrary DFNs based on a PDE-constrained optimization reformulation of the problem. The proposed approach aims at allowing the use of non-conforming grids, thus facilitating the meshing process. The exact fulfillment of the matching conditions at fracture intersections is replaced by the minimization of a properly defined functional. The minimization process is constrained by the local state equations. After a suitable space discretization, the overall problem is reformulated as a quadratic programming problem with linear equality constraints. Within such reformulation, we are able to mesh each fracture independently of the other fractures, thus totally circumventing any problem in the mesh generation. Furthermore, the method is naturally conceived in a fracture-oriented way, and decoupled computations on the fractures are envisaged.

Due to the very large scales encountered, iterative solvers and related preconditioning issues are a must, and parallel computing is in order. Focusing on the linear algebra issues related to the method, we will discuss robustness and efficiency of the approach on realistic DFNs, with a large heterogeneity in fracture dimensions, distance, and angles formed by fracture intersections. Scalability performances of a tailored parallel implementation will be also discussed.

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# The simplification of Spectral Projected Gradient method for solving quadratic programs

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The numerical solution of many engineering problems leads to the problem of minimizing a strictly convex quadratic function subject to a given set of inequality constraints. The applications that will benefit from the development of optimal algorithms for solving such an optimization problem are, for instance, the linear elasticity contact problems or the simulation of granular dynamics.

Birgin, Martínez, and Raydan [1] developed for solving such problems a new projected gradient descent method based on the combination of projected Barzilai-Borwein method [2] with additional Grippo-Lampariello-Lucidi line-search technique [3] to enforce the convergence using generalized Armijo condition. This Spectral Projected Gradient method (SPG) is successfully used by many authors for solving more general optimization problems than quadratic programs; however, authors in original paper demonstrates the efficiency solving box-constrained quadratic programming problems.

In our contribution, we examine the behaviour of SPG for solving optimization problems with strictly convex quadratic cost function. Using our observations, we are able to suggest a simple way how to simplify the whole algorithm. The original SPG performs several function value evaluations whereas our modification contains only one multiplication by Hessian matrix per iteration.

The performance of the algorithm is demonstrated and compared with original SPG on the solution of practical benchmark.

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**Title:** Constrained polynomial optimization on commutative and non-commutative variables

**Speaker:** prof. Janez Povh

**Abstract:**

Polynomial optimization problems are in the intersection between mathematical programming society and real algebraic geometry.

Advances in each area accelerates development in the other area and vice versa.

In the talk we will first present state of the art in commutative polynomials optimization, especially topics related to semidefinite programming approximation hierarchies.

The main part of the talk will be devoted to non-commutative extensions, more precisely to constrained trace and eigenvalue optimization of noncommutative polynomials.

We will present Lasserre's relaxation scheme for trace and eigenvalue optimization based on semidefinite programming (SDP) and demonstrate its convergence properties.

Finite convergence of these relaxation schemes is governed by flatness, i.e., a rank-preserving property

for associated dual SDPs. If flatness is observed, then

optimizers can be extracted using the Gelfand-Naimark-Segal construction and the Artin-Wedderburn theory

verifying exactness of the relaxation.

To enforce flatness we employ a noncommutative version of the randomization technique championed by Nie.

The implementation of these procedures in our computer algebra system NCSOStools will be presented together with several examples illustrating our results.



13<sup>th</sup> EUROPT Workshop on Advances in Continuous Optimization

**Title:** Augmented Lagrangian methods for nonlinear programming with possible infeasibility

**Speaker:** Leandro F. Prudente

**Authors:** M.L.N. Gonçalves, J.G. Melo, L.F. Prudente

**Abstract:** We consider a nonlinear programming problem for which the constraint set may be infeasible. We propose an algorithm based on a large family of augmented Lagrangian functions and, accepting inexact global solutions of the subproblems, analyze its convergence properties taking into account the possible infeasibility of the problem. In a finite number of iterations, the algorithm stops detecting the infeasibility of the problem or finds an approximate feasible/optimal solution with any required precision. We present some numerical experiments illustrating the applicability of the algorithm for different Lagrangian/penalty functions proposed in the literature.

Title: Convex Euclidean Distance Embedding for Collaborative Position Localization with NLOS Mitigation

Authors: Chao Ding and Hou-Duo Qi

Abstract: One of the challenging problems in collaborative position localization arises when the distance measurements contain Non-Line-Of-Sight (NLOS) biases. Convex optimization has played a major role in modelling such problems and numerical algorithm developments. One of the successful examples is the Semi-Definite Programming (SDP), which translates Euclidean distances into the constraints of positive semidefinite matrices, leading to a large number of constraints in the case of NLOS biases.

In this paper, we propose a new convex optimization model that is built upon the concept of Euclidean Distance Matrix (EDM).

The resulting EDM optimization has an advantage that its Lagrangian dual problem is well structured and hence is conducive to algorithm developments. We apply a recently proposed 3-block alternating direction method of multipliers to the dual problem and tested the algorithm on some real as well as simulated data of large scale. In particular, the EDM model significantly outperforms the SDP model and several others.

## **Single period portfolio selection models with transaction costs and initial holdings**

Marius Rădulescu and Constanța Zoie Rădulescu

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We extend Markowitz's portfolio selection model to include transaction costs in the presence of initial holdings for the investor. Our approach is new. We obtain a multiobjective optimization model which includes complementarity constraints. The objective functions are risk and expected return. Starting from the multiobjective optimization model we formulate several single objective models.

The single period models include a minimum risk model and a maximum expected return model.

Our portfolio selection models include complementarity constraints. This type of constraints increases the difficulty of the problems, which now enter in the category of combinatorial optimization problems.

The set of feasible solutions for the problems from the above mentioned class is the union of a set of convex sets but it is no longer convex. We build equivalent mixed integer models for the single period portfolio selection models. We provide heuristic algorithms for finding solutions of portfolio selection models. Several numerical results are discussed.

# A linesearch derivative-free method for bilevel minimization problems

S. Lucidi\*

S. Renzi\* \*

March 19, 2015

## Abstract

In this work we consider bilevel optimization problems which are programs with a particular nested structure. In such structure a problem, called lower level or inner problem, is nested inside the feasible region of another problem, called upper level or outer problem. When, for any fixed upper level choice, the lower level optimal solution is not uniquely determined then the outer minimization can be ill-defined. To address this pathology there are two main approaches: Optimistic and Pessimistic. In order to overcome this possible issue, we focused on bilevel programs with a supposed strictly convex lower level problem. In this framework we propose a new linesearch derivative-free resolution algorithm. Derivative Free (or Direct Search) methods do not require knowledge or calculation of derivative and for these reasons they are widely used to solve real world problems. The idea underlying this work is to use a Derivative Free Non-Smooth algorithm that, for each trial point which it explores, will compute the solution of the lower level problem. Roughly speaking, at each iteration of the algorithm an appropriate set of direction is explored in order to find a new point where the objective function value is sufficiently lower than the one assumed in the current optimal point. Under suitable assumptions it is possible to prove that an accumulation point of the sequence produced by the algorithm is a stationary point of the considered problem. With the aim to evaluate the performance of the proposed algorithm different numerical tests have been performed. The results of the preliminary numerical experience showing a possible practical interest in the proposed approach.

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# Strategic gaming analysis for cement industry: a bilevel approach

E. Allevi\*    A.J. Conejo<sup>†</sup>    G. Oggioni<sup>‡</sup>    R. Riccardi<sup>§</sup>    C. Ruiz<sup>¶</sup>

## Abstract

This paper investigates the equilibria reached by a number of strategic producers in the cement sector through a technological representation of the market. We present a bilevel model for each producer that characterizes its profit maximizing behavior. In the bilevel model, the upper-level problem of each producer is constrained by a lower-level market clearing problem representing cement trading and whose individual objective function corresponds to social welfare. Replacing the lower level problem by its optimality condition renders a Mathematical Program with Equilibrium Constraints (MPEC). Then, all strategic producers are jointly considered. Representing their interaction requires solving jointly the interrelated MPECs of all producers, which results in an Equilibrium Problem with Equilibrium Constraints (EPEC).

A parametric analysis concerning cost and demand fluctuations has been conducted.

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## **On a smoothed penalty-based artificial fish swarm algorithm for global optimization**

Ana Maria A.C. Rocha, M. Fernanda P. Costa and Edite M.G.P. Fernandes

This paper presents a coercive smoothed penalty framework for nonconvex constrained global optimization problems. At each iteration, the penalty framework requires an approximation to the global minimizer of the smoothed penalty function. This subproblem is solved by a stochastic population-based algorithm, known as the artificial fish swarm (AFS). We prove that, in the limit, the convergence to an approximate global minimizer of the real-valued smoothed penalty function is guaranteed with probability one, using the limiting behavior of Markov chains. Preliminary numerical experiments show that the presented penalty algorithm based on the AFS algorithm gives very competitive results when compared with other stochastic penalty-based methods.

# From first to second-order quality measures in direct-search methods

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Direct-search methods are one of the main classes in derivative-free optimization, due to their simplicity and well established convergence results. They proceed by iteratively looking for improvement along some vectors or directions. In presence of smoothness, a convergence proof can be derived by studying at each iteration the cosine of the minimum angle between those vectors and the direction of steepest descent. Provided this (cosine) measure remains bounded away from zero and the algorithm enforces a sufficient decrease condition to move towards a better point, a first-order global convergence property is obtained as well as a worst-case complexity bound.

In this talk, we develop a second-order study of a general class of direct-search methods. A measure of second-order criticality inspired by the Rayleigh quotient is presented. Using this measure, a weak second-order global convergence result is derived for any method in the class. Extensions of this result to ensure true second-order global convergence are discussed, among which a method using approximate Hessian eigenvectors as directions. The means to achieve such a global convergence are costly to ensure (a fact well reflected in the worst-case complexity analysis) but seem necessary as some pathological numerical instances indicate.

**Gizem Sağol, E. Alper Yıldırım**

## **Copositive Optimization Based Bounds on Box Constrained Quadratic Optimization**

Box constrained quadratic optimization problems (BoxQPs) can be formulated as a linear optimization problem over the cone of completely positive matrices in several different ways. We consider two alternative formulations. We study the sequences of upper and lower bounds on the optimal value of a BoxQP arising from two hierarchies of inner and outer polyhedral approximations for both of these formulations.



# A Class of Convergent Parallel Algorithms for SVMs Training

Simone Sagratella, Andrea Manno, Laura Palagi

March 27, 2015

The training of Support Vector Machines (SVMs) may be a very difficult task when dealing with very large datasets. In fact memory requirement and time consumption of SVMs training algorithms grow rapidly with the increase of data size. To overcome these drawbacks a lot of parallel algorithms were implemented, but they lack of convergence properties. In this work we propose a general parallel decomposition scheme for training SVMs and we state its asymptotical global convergence under suitable conditions. We outline how these assumptions can be satisfied in practice and we suggest various specific implementations exploiting the adaptable structure of the algorithmic model. In particular we illustrate the versatility of our general decomposition scheme by specializing it to both sequential and parallel algorithms.

# Shape Optimization Method for Designing Stationary Plasma Thrusters

Satafa SANOGO      Frédéric MESSINE      Carole HÉNAUX

March 26, 2015

## Abstract

The aim of this paper is to propose algorithms for designing optimal magnetic circuits based on Shape Optimization (SO) method. In our work, we are interested in the design of Stationary Plasma Thrusters (SPTs) which are ion thrusters and are used in many space operations boarding in satellites nowadays. Through many researches, it was shown that the performance and efficiency of these devices depend strongly on the topology and the shape of their electromagnetic circuits. Then, for a requested performance a specific spatial distribution of the electromagnetic field is imposed in some points of the engine and the purpose is to find an electromagnetic circuit capable to produce this target field. Thus, the considered problems are inverse ones. They are formulated as SO problems with Boundary Variation Approach (BVA). In this paper, the shape derivative is computed with an Adjoint Variable Method (AVM) which makes it possible to apply gradient based algorithms (which are most efficient in general). We have written a SO routine using the Finite Element software **FreeFem++**. Some numerical experiments of design process of a STP magnetic circuit are performed to validate our approach.

## Keywords

Electrical Thruster, Electromagnetism, Inverse Problem, Topology Optimization, Shape Optimization, Adjoint Variable Method, Shape derivative.

**Title:**

On Finding a Generalized Lowest Rank Solution to a Linear Semi-definite Feasibility Problem

**Author:**

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**Abstract:**

We generalize the affine rank minimization problem and the vector cardinality minimization problem and it is found that the resulting generalized problem can be solved by solving a sequence of continuous concave minimization problems. In the case of the vector cardinality minimization problem, it can be solved exactly by solving the continuous concave minimization problem.

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# On the relation between conjugate gradient and quasi-Newton algorithms

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## ABSTRACT

In the paper the new formulation of the Broyden restricted convex class of updates involving oblique projections found recently by Stachurski is further exploited. First the new proof of the inheritance of the directions conjugacy is presented when the methods are applied to the strictly convex quadratic problems and the exact directional minimization is used. This analysis leads to another formulation of the quasi-Newton updates. Their construction requires only some amount of differences of unknowns, values of the scalar products with the corresponding differences of the goal function derivatives and the last difference of the derivatives itself. Appropriate selection of the coefficients in the studied class of methods generates the same directions as in the recently popular memory gradient methods. The methods considered may represent a good alternative to the limited – memory BFGS method with an advantage that it doesn't require the precise formulation of the approximation of the inverse of the hessian matrix. Some preliminary computational benchmarking results will be reported.

We view the derived formulation of the quasi-Newton updates as an answer to the question raised by Andrei concerning the relation between conjugate gradients and quasi-Newton methods.

# Coercive polynomials and their Newton polytopes

Tomáš Bajbar<sup>1</sup> and Oliver Stein<sup>2</sup>

Many interesting properties of polynomials are closely related to the geometry of their Newton polytopes. In this talk we analyze the coercivity on  $\mathbb{R}^n$  of multivariate polynomials  $f \in \mathbb{R}[x]$  in terms of their so-called Newton polytopes at infinity. In fact, we introduce the broad class of so-called gem regular polynomials and characterize their coercivity via conditions solely containing information about the geometry of the vertex set of the Newton polytope at infinity, as well as sign conditions on the corresponding polynomial coefficients.

For all other polynomials, the so-called gem irregular polynomials, we introduce sufficient conditions for coercivity based on those from the regular case. For some special cases of gem irregular polynomials we establish necessary conditions for coercivity, too. Using our techniques, the problem of deciding the coercivity of a polynomial can be reduced to the analysis of its Newton polytope at infinity. We relate our results to the context of the polynomial optimization theory and the existing literature therein, and we illustrate our results with several examples.

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# Accelerated Random Search

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We study randomized accelerated methods for convex optimization, i.e. schemes where the search directions are not the negative gradients, but samples from a discrete or continuous probability distribution.

Whilst the classical Gradient descent converges at rate  $O(k^{-1})$ , it is well-known that optimal schemes converge at rate  $O(k^{-2})$ . In 1983, Nesterov proposed a framework that allows to construct optimal schemes by means of approximations, so called estimate sequences. Recently, Nesterov (2011,2012) showed that the same ideas can be used to accelerate randomized schemes, where either the search directions are sampled from an isotropic normal distribution or the set of coordinate directions. Lee and Sidford (2013) extended the estimate sequence framework to this setting, but they restricted themselves to distributions over coordinate directions. To achieve the optimal convergence rate, the choice of the step sizes is crucial. The two aforementioned schemes therefore require access to either directional derivatives or to gradient oracles (random variables whose expectation equals the gradient of the objective function at the query point).

In this work, we study the impact of the step sizes on the convergence rate. The analysis is not limited to discrete or isotropic distributions, but also applies to certain non-isotropic continuous distributions. We consider (i) inexact gradient oracles and (ii) (gradient-free) inexact line search oracles. These modified schemes can still accelerate, but they do not necessarily attain the optimal rate, that is, they converge at rate  $O(k^{-\alpha})$ , for  $1 \leq \alpha \leq 2$ . These modifications are motivated by two concrete examples: (i) high-dimensional optimization, where directional derivatives might only be computed approximately for efficiency reasons, and (ii) derivative-free optimization, where no gradient information is available but only function values.

## **A New Algorithm for the Optimal Design of Anisotropic Materials**

M. Stingl, Department of Mathematics, Friedrich-Alexander-University of  
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A new algorithm for the solution of optimal design problems with control in parametrized material coefficients is discussed. The algorithm is based on the sequential convex programming idea, however, in each major iteration a model is established on the basis of the parametrized material tensor. The potentially nonlinear parametrization is then treated on the level of the sub-problem, where, due to block separability of the model, globally optimal solutions can be computed. Although global optimization of non-convex design problems is in general prohibitive, a smart combination of analytic solutions along with standard global optimization techniques leads to a very efficient algorithm for the most relevant material parametrizations. Theoretical properties of the algorithm are discussed. The effectiveness of the algorithm in terms of computation time as well as quality of the solution with respect to global lower bounds is demonstrated by a series of numerical examples. Examples range from free material optimization problems via parametric and discrete material optimization problems (e.g. optimal orientation problems) to two-scale material design.

## **CQ-free optimality conditions for convex SIP problems with finitely representable compact index**

Authors: Kostyukova O.I. (Institute of Mathematics, National Academy of Sciences of Belarus) and Tchemisova T.V. (Center for Research and Development in Mathematics and Applications, Department of Mathematics, University of Aveiro)

**Abstract:** We study optimality conditions for convex Semi-Infinite Programming problems with arbitrary index sets defined by finite numbers of nonlinear inequalities. The main result consists in formulation and proof of the implicit and explicit optimality conditions that do not use constraint qualifications (CQ-free). We show that these optimality conditions are more efficient than that known from the literature. We apply the results obtained to some special cases of the convex SIP problems and show how new optimality conditions for these problems can be formulated using the main results of the paper.



# Critical objective size and calmness modulus in linear programming\*

M.J. Cánovas<sup>†</sup> · R. Henrion<sup>‡</sup> · J. Parra<sup>†</sup> · F.J. Toledo<sup>†</sup>

## Abstract

This talk introduces the concept of critical objective size associated with a linear program in order to provide operative point-based formulas (only involving the nominal data, and not data in a neighborhood) for computing or estimating the calmness modulus of the optimal set (argmin) mapping under uniqueness of nominal optimal solution and perturbations of all coefficients. Our starting point is an upper bound on this modulus given in [1]. In this talk we show that this upper bound is attained if and only if the norm of the objective function coefficient vector is less than or equal to the critical objective size. This concept also allows us to obtain operative lower bounds on the calmness modulus. We analyze in detail an illustrative example in order to explore some strategies that can improve the referred upper and lower bounds.

**Keywords.** Variational analysis · Calmness · Linear programming

**Mathematics Subject Classification:** 90C31, 49J53, 49K40, 90C05.

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## **Chordal graphs and sparse semidefinite optimization.**

Lieven Vandenberghe

Chordal graphs have a rich history in several disciplines, including combinatorial optimization, statistics, signal processing, machine learning, sparse matrix algebra, and nonlinear optimization. This lecture will give a survey of some of the theory and applications of chordal graphs. The emphasis will be on techniques developed in the literature on sparse Cholesky factorization, and on sparse matrix algorithms formulated as recursions on elimination trees, supernodal elimination trees, or clique trees associated with the sparsity graph. The best known example is the multifrontal Cholesky factorization algorithm, but similar algorithms exist for related problems, for example, the computation of the partial inverse of a sparse positive definite matrix, positive semidefinite and Euclidean distance matrix completion problems, and the evaluation of gradients and Hessians of logarithmic barriers for cones of sparse positive semidefinite matrices and their dual cones. The techniques will be illustrated with applications to interior-point algorithms and decomposition methods in sparse semidefinite optimization.

# Influence of matrix reordering on the performance of the iterative methods for solve linear systems arising from interior point methods.

Marta Velazco<sup>1</sup>, Daniele Silva<sup>2</sup> and Aurelio Oliveira<sup>3</sup>

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The searching directions in interior-point method are computed through the solution of one or more linear systems. The solution of these systems is the most expensive step of these methods. Such systems are indefinite and can be reduced to a smaller positive-definite system, i.e. normal equations. For some classes of large-scale problems, the use of direct methods is prohibiting because of storage and running-time limitations. In such situations, iterative approaches are recommended.

The performance of implementations using iterative methods depends on the choice of an appropriate preconditioner, in particular for interior point methods; the linear system becomes highly ill-conditioned as an optimal solution of the problem is approached. Recently, a two phase hybrid preconditioner was proposed that uses the controlled Cholesky factorization during the initial iterations and in the remaining ones adopts the splitting preconditioner.

In context of direct methods to solve the interior point systems, sparse matrix reordering technique has been extensively adopted. For iterative methods, the use of matrix reordering can also improve the time of computing preconditioner based on incomplete Cholesky factorization and reducing inner iterations.

This work analyzes the influence of sparse matrix reordering over performance of preconditioner conjugate gradient method for solve normal equations systems of interior point methods using the hybrid preconditioner. The reordering of reverse Cuthill-McKee heuristics, the Sloan algorithm and minimum degree are analyzed in this study. Numerical experiments indicate that these heuristics accelerate the convergence of iterative methods and improve the solution time.

# Robust Multiobjective Linear Optimization

José Vicente-Pérez\*

## Abstract

In this talk we deal with multi-objective linear programming problems in the face of data uncertainty both in the objective function and the constraints. We first provide a formula for the radius of robust feasibility guaranteeing constraint feasibility for all possible scenarios within a specified uncertainty set under affine data parametrization. We then establish dual characterizations of robust solutions of our model that are immunized against data uncertainty by way of characterizing corresponding solutions of its robust counterpart, and we present robust duality theorems. Besides that, we show numerically tractable optimality conditions for minmax robust weakly efficient solutions, i.e., the weakly efficient solutions of the robust counterpart. We also consider highly robust weakly efficient solutions, i.e., robust feasible solutions which are weakly efficient for any possible instance of the objective matrix within a specified uncertainty set.

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# An Improved Two-Stage Optimization-Based Framework for Unequal-Areas Facility Layout Problem

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## Abstract

The unequal-areas facility layout problem is concerned with finding the optimal arrangement of a given number of non-overlapping indivisible departments with unequal area requirements within a facility. We present an improved optimization-based framework for efficiently finding competitive solutions for this problem. The framework is based on the combination of two mathematical optimization models. The first model is a nonlinear approximation of the problem that establishes the relative position of the departments within the facility, and the second model is an exact convex optimization formulation of the problem that determines the final layout. Aspect ratio constraints on the departments are taken into account by both models. Our computational results show that the proposed framework is computationally efficient and consistently produces competitive, and often improved, layouts for well-known instances from the literature as well as for new large-scale instances with up to 100 departments.

# Polyhedral Approximations of Completely Positive Optimization Problems

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## Abstract

Completely positive optimization deals with the optimization of a linear matrix function over an affine subspace of the cone of completely positive matrices. Recently, it has been shown that every quadratic optimization problem with a mix of binary and continuous variables can be formulated as an instance of a completely positive optimization problem. Therefore, despite the convex nature of this class of optimization problems, the cone of completely positive matrices is computationally intractable. We discuss polyhedral approximations of completely positive optimization problems. We present our results on the quality of these polyhedral approximations on certain classes of quadratic optimization problems.

# A New Proposal for the Approximate Solution of the Normal Equations in Primal-Dual Interior Point Methods

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## ABSTRACT

The main computational operation of an Primal-Dual interior-point method (IPM) for linear programming problem is the solution of a Newton method system [1]. In real applications this system often has large dimension and a high degree of sparsity and its solution is the most expensive method step. For this reason, much effort has been employed by researchers in the attempt to find methods for solving this system more efficiently.

This system can be reduced to other system which is symmetric positive definite, called normal equations system. The normal equations solution using Cholesky factorization is preferable whenever this factorization is not very expensive. When the Cholesky factorization is very expensive, iterative methods for solving the normal system becomes more appropriate. In this context, an approach could be used is the preconditioned conjugate gradient method with the hybrid preconditioner composed by the controlled Cholesky factorization (CCF) in early iterations and the splitting preconditioner in the later iterations [2].

The CCF is an incomplete Cholesky factorization, where we obtain a lower triangular matrix  $L$  less dense than the Cholesky factor, such that  $ADA^t = LL^t - R$ , where  $R$  is the remainder matrix. The fill-in at each  $L$  column is controlled by a parameter  $\eta$  in the following way: given  $L_j$  column, only the  $k_j = \eta + n_j$  largest nonzeros entries will be kept in the column, where  $n_j$  is the column  $j$  nonzero entries of the normal equation system matrix [3].

The aim of our work is to improve the Primal-Dual IPM reducing processing time and/or the storage required in each iteration. This will be done by replacing the Cholesky factorization by CCF. In early iterations, we can obtain an approximate direction of the original one, adopting a CCF parameter value such that the matrix obtained in the factorization is very sparse, speeding up the linear system solution. In later iterations we compute CCF factorizations closer to the complete Cholesky factorization, in such a way that the method achieve convergence. Numerical experiments show that this approach obtains competitive results.

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## **Solving Ill-posed Bilevel Programs**

Alain B. Zemkoho

Ill-posed bilevel programs are bilevel optimization problems admitting multiple lower-level solutions for some upper-level parameters. In this talk, we show that the original optimistic version of the problem can be the base to solve all the other models including the pessimistic and standard optimistic problems. Next, we establish an equivalence between this model and a certain set-valued optimization problem, thus simplifying the problem by reducing its level from three to two. Finally, we show how the aforementioned set-valued optimization problem can be solved in practice.



# Interval arithmetic and copositivity detection

Julius Žilinskas

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## **Abstract**

Interval arithmetic is a valuable tool in numerical analysis and global optimization. Interval arithmetic operates with intervals defined by two real numbers and produces intervals containing all possible results of corresponding real operations with real numbers from each interval. Interval methods for global optimization detect subregions where there are no better values of objective function than a known value, where the objective function is monotone, etc. Detection of copositivity is important in combinatorial and quadratic optimization. Several proposed algorithms for copositivity detection use simplicial partitions. However, interval arithmetic is not directly applicable in the case of simplicial subregions. In this talk we investigate possibility to use interval arithmetic in detection of copositivity.