# Sir Michael Atiyah — Fields medal · Bridge to Physics



### *Fields medal*

Atiyah was awarded the Fields medal at the 1966 International Congress of Mathematicians (Moscow, Soviet Union) with the following citation:

> Michael Francis Atiyah (Oxford University). *Did joint work with Hirzebruch in K-theory; proved jointly with Singer the index theorem of elliptic operators on complex manifolds; worked in collaboration with Bott to prove a fixed point theorem related to the "Lefschetz formula".*

The other Fields medallists in 1966 were:

> Paul Joseph Cohen (Stanford University). *Used the technique called "forcing" to prove the independence in set theory of the axiom of choice and of the generalized continuum hypothesis. The latter problem was the first of Hilbert's problems of the 1900 Congress.*

> Alexander Grothendieck (University of Paris) *Built on work of Weil and Zariski and effected fundamental advances in algebraic geometry. He introduced the idea of K-theory (the Grothendieck groups and rings). Revolutionized homological algebra in his celebrated "Tohoku paper"* [3]

> Stephen Smale (University of California, Berkeley). *Worked in differential topology where he proved the generalized Poincaré conjecture in dimension n≥5: Every closed, n-dimensional manifold homotopy-equivalent to the n-dimensional sphere is homeomorphic to it. Introduced the method of handle-bodies to solve this and related problems.*

#### *Sources*

The text on this poster is extracted from [1], with some slight changes to fit the narrative style and additions selected from the International Mathematical Union (IMU) web page. The pictures of Hitchin and Witten have been obtained from their web pages, the picture of Penrose from his MacTutor biography, and the others from the IMU web page.

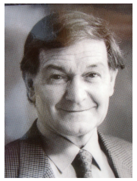
[1] M. Atiyah: *Autobiography* (text written for the Abel Prize Committee, 2004).
[2] E. Witten: *Michael Atiyah and the Physics/Geometry Interface*. Asian J. Math., Volume 3, No. 1, pp. lxi-lxiv.
[3] *Sur quelques points d'algèbre homologique*, Tohoku Math. J.9, 119–221 (1957).

### *Geometry and Physics*

In 1973 Atiyah returned to Oxford. While he had no formal teaching duties, he supervised, over the years, a string of talented research students who also influenced his research, basically focussed on the interaction between geometry and physics. By the late 70's the interaction between geometry and physics had expanded considerably. The index theorem became standard form for physicists working in quantum field theory, and topology was increasingly recognized as an important ingredient.

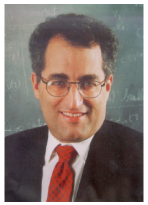
In those endeavours, he also collaborated with many other colleagues.

Nigel Hitchin, who had moved to Princeton with Atiyah and then returned to Oxford, is now Savilian Professor of Geometry at Oxford. They have collaborated on several topics, most notably instantons and magnetic monopoles. Concerning the latter, there is an interesting video produced by IBM in 1988 that displays their puzzling low energy scattering behaviour (it can be accessed from Hitchin's Web page).



Sir Roger Penrose. A Cambridge contemporary of Atiyah, they met again in 1973. From the interaction with Atiyah it soon appeared that the complicated contour integrals in Penrose's twistor theory could be reinterpreted in terms of sheaf cohomology. This established a key bridge between Atiyah's and Penrose's groups. At present, Penrose is an Honorary Professor of Mathematics at the University of Oxford.

The results on instantons and monopoles opened doors, for Atiyah and his group, to a wider physics community. They also led to the spectacular results of Simon Donaldson on 4-dimensional geometry, one of the highlights of 20th century mathematics. Donaldson was awarded the Fields Medal at the 1986 International Congress of Mathematicians (Berkeley, California, USA) *primarily for his work on topology of four-manifolds, especially for showing that there is a differential structure on euclidian four-space which is different from the usual structure.** At present Donaldson is Royal Society Research Professor at the Imperial College in London.



Edward Witten. For over thirty years he has been recognised as the driving force among theoretical physicists exploring the frontiers of their subject. He has provided mathematicians with an entrée to theoretical physics which is remarkable in its richness and sophistication. Atiyah expresses that he was "fortunate to get to know Witten fairly early in his career while he was a Junior Fellow at Harvard" (1977) and that he "learned a great deal from him".

* The report on Donaldson's work was written by Atiyah and can be found in the proceedings of ICM86 (or in Volume 6 of Atiyah's Collected Works).

Witten was awarded the Fields Medal at the 1990 International Congress of Mathematicians (Kyoto, Japan). The report on his work, a short masterpiece, was written by Atiyah (proceedings of ICM90, or in Volume 6 of Atiyah's Collected Works). Here are a few quotations:

> Although he is definitely a physicist, his command of mathematics is rivalled by few mathematicians, and his ability to interpret physical ideas in mathematical form is quite unique.
>
> His 1984 paper on supersymmetry and Morse theory is obligatory reading for geometers interested in understanding modern quantum field theory.
>
> He made the important observation that the $\eta$-invariant of Dirac operators (introduced by Atiyah, Patodi and Singer) is related to the adiabatic limit of a certain anomaly.
>
> It was a considerable surprise when Witten outlined a much simpler proof [than that by Schoen and Yau] of the positive mass conjecture in General Relativity [...] using spinors and the Dirac operator.
>
> One of the remarkable aspects of the Geometry/Physics interaction of recent years has been the impact of quantum field theory on low-dimensional geometry (of 2, 3 and 4 dimensions). Witten has systematized this whole area by showing that there are, in these dimensions, interesting *topological* quantum field theories.
>
> Witten's approach is extremely powerful and flexible, suggesting a number of important generalizations of the theory wich are currently being studied and may prove to be important.
>
> So far his insight has never let him down and rigorous proofs, of the standard we mathematicians rightly expect, have always been forthcomming.

Conversely, Witten acknowledges a deep and sustained influence of Atiyah on his research in [2]. Here are a few quotations from this wonderful paper:

> Theoretical physicists had certainly not yet realized that the gauge theory revolution had created a situation in which it would be necessary and worthwhile to develop a greater mathematical sophistication than we were accustomed to. Atiyah and other mathematicians [...] played an important role in the process.
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> Albert Schwarz showed [in 1976] that some of the ingredients of the solution [of the so-called $U(1)$ problem, using instantons] were best understood in terms of the Atiyah-Singer index theorem. [...] In the theoretical physics environment of those days, [... it] was way beyond the prevailing level of mathematical sophistication.
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> In 1987 [...] Atiyah hoped that a quantum field theory with Donaldson polynomials as correlation functions and Floer groups as the Hilbert spaces could somehow be constructed by physics methods. [Witten showed that it was the case at the end of that year]
>
> [In 1988] Atiyah considered a major piece of unfinished business to understand the Jones polynomial in terms of quantum field theory. [Witten solved the problem shortly, with clues provided by Atiyah, and ...] This work relating the Jones polynomial to Chern-Simons was a turning point in my career.
>
> I have tried to recount a few of the highlights of my scientific interactions with Michael Atiyah, and to convey a little of the role he played in encouraging us to study quantum field theory from new points of view. We had to learn a lot of lessons before taking these new perspectives seriously. Atiyah, along with colleagues such as Bott and Is Singer, played an important role in teaching some of these lessons to the physics world. Atiyah has always believed intuitively that the study of quantum field theory as a tool in geometry had to be integrated with the study of more "physical" aspects of quantum field theory. This was one of the hardest lessons for me personally to learn.