Iron Rings, Doctor Honoris Causa Raoul Bott, Carl Herz, and a Hidden Hand

P. Robert Kotiuga

ABSTRACT. The degree of Doctor of Sciences, honoris causa, was conferred on Raoul Bott by McGill University in 1987. Much of the work to make this happen was done by Carl Herz. Some of the author's personal recollections of both professors are included, along with some context for the awarding of this degree and ample historical tangents. Some cultural aspects occurring in the addresses are elaborated on, primarily, the Canadian engineer's iron ring. This paper also reprints both the convocation address of Raoul Bott and the presentation of Carl Herz on that occasion.

Introduction

Raoul Bott needs no introduction in this volume. However, reprinting his address at the 1987 McGill convocation both gives some insight into the effort to award him an Honorary Doctorate in Mathematics from McGill, and a context to develop some less than mathematical themes, unashamedly from the point of view of an electrical engineer who enjoys the historical aspects of his discipline. Early on I was tipped off that Carl Herz was behind the effort, and the memory of Prof. Herz made me realize that I had to follow the trail like a hound. The result complements more technical presentations and anecdotes pertaining to Montreal[1]. I am grateful to Candace Bott for digging up her father's commencement[2] address, and to Dominique Papineau of McGill University who showed up in Boston with a complete file pertaining to the awarding of the honorary degree from McGill's archives. The support of various McGill faculty who listened to me think aloud is much appreciated, namely Peter Caines, Jacques Hurtubise, Joachim Lambek, and Peter Russell.

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[1]For anecdotes from Bott's years in Montreal see [**Tu**] and Candace Bott's remarks in this volume.

[2]Although "convocation" and "commencement" have different meanings in general, in the context of a university graduation ceremony they have the same meaning in Canada and in the USA respectively. For the purposes of this article, they are used interchangably.

How I got to know Raoul Bott

I can't remember who first connected Raoul Bott to McGill Engineering in my mind; most likely it was Peter Caines, Robert Hermann or Carl Herz. However, I do remember attending his talk in the Physics department at McGill in, I believe, 1982. These days, if the fact that I dragged my wife to be to the talk is mentioned, and that she coped with my enthusiasm with a sense of humor, my kids will kindly remind me of all my thrifty ideas for a good time! During those years I spent many hours "teething" on the book of Bott and Tu, and as a NSERC post-doc in the MIT Mathematics department in 1985 I finally got the opportunity to audit a course given by Raoul Bott. Not only was his mathematics entrancing, but we were both McGill engineers! During the first lecture he spotted my iron ring. I introduced myself afterwards, and we talked about a variety of things. Before long, I made a habit of auditing every course he taught. About a year later, he told me that he would be the recipient of an honorary doctorate from McGill — forty years after McGill wouldn't have him as a graduate student in the mathematics department (he was unwilling to complete a second undergraduate degree in mathematics). Past history aside, he seemed genuinely honored, but he didn't quite know who to share this news with. Reading the convocation speech, I now see it as his way to make peace with history and a means to repackage it constructively for graduates forty years younger than himself. Stepping back from the ceremony from decades ago, the reader is invited to read his "Autobiographical Sketch"[**BoA**].

I clearly enjoyed his lecturing style as well as the attention and discipline he demanded in the classroom. It certainly contrasted with the convocation speech. In the first lecture, he encouraged students to ask questions and claimed that he liked "stupid questions" because he could answer them. He was very serious about this and if we didn't realize it initially, we eventually learned that he put the bar very high, and even higher for himself: His answers to questions were always more profound than the original questions and he once walked out of his own lecture in frustration because he didn't like it! That was dramatic, baffling, and unexpected — especially since we knew he liked to think on his feet. In this particular case, he reappeared the next class, his arguments were impeccably clear and elegant, and his credibility was not only restored, it soared! Going beyond the classroom, Bott's mentoring of graduate students is the source of legend. This volume has plenty of testimony from his students, and the reminiscences by Robert MacPherson [**MP**] testify to his high expectations. The convocation speech is very different in that it demonstrates his ability to connect with an audience that has never seen him and most likely will never see him again. Before attempting to advise graduates, he warms up the audience by telling of the pivotal event in his professional life and saying: "I tell you all this only in part as a jest." Only when the stage is set does he give the essence of his address credibly and in a few sentences. The message appears in a flash after saying "But my time is up!". The style mirrors his approach to giving a colloquium talk.

Bott enjoyed cultivating certain habits which were best left unmentioned in the convocation speech. For instance, he lectured at 8:30 a.m. in order to have a flexible day after 10:00 a.m.! Graduate students felt this cramped both their style and sleeping habits, and this is where some of Bott's Old World sensibilities kicked in when they dared to doze off in class. He could toss chalk and have it land on the table inches from the sleeping student's face, startling them. He clearly relished

doing so. The memorable line which would meet the startled face changed with every successive offense. From "didn't want you to miss anything" to "how much is tuition at Harvard?" to "who pays your tuition?" In Old World style, this was all for the benefit of the student and there was little room for self-preservation. These days it isn't easy for a professor to be respected for doing this in a private university where students can feel like paying customers once the tuition bill is settled. Somehow Bott was consistently more mischievous than the students, and got away with it. Clearly he had extensive experience testing his teachers and this experience always gave him the upper hand in the classroom.

As for the impact Bott's lectures made on me, I'd be treading on thin ice if I tried to say why they were fantastic. Loring Tu says that[3] Victor Guillemin, at a conference celebrating his 60th birthday, proudly announced that he took twelve courses from Bott, and to Loring's chagrin, he could only list eleven. Clearly, I am in no position to speak with authority about Bott's lectures! In my case, I loved his lecture style, the lecture material, and I felt a definite kinship since I could ask tangentially related questions after class and consistently get profound answers. There is perhaps one personal anecdote I can add to the many that I've heard. One day in class, after Bott explained the set-up of the Lefschetz fixed point theorem in terms of the transverse intersection of the graph of a map from a manifold to itself with the graph of the identity map, he claimed that, by duality, the Lefschetz number could be easily computed by picking a basis for integral cohomology, pulling back by the appropriate maps, taking wedge products with Poincare duals and integrating. When he claimed that it reduced to basic matrix algebra involving the induced automorphisms on cohomology groups which an engineer could do, the class just didn't make the type of eye contact he was hoping for. At that point he called me up to the board and told me to fill in the details of the calculation! As I (methodically) wrapped up the calculation, he identified me as an engineer, emphasized that budding topologists shouldn't shy away from such concrete calculations, and took satisfaction in the fact that he made his point. In my mind he reinforced the fact that Daniel Quillen's thesis advisor could make us think functorially while, as a student of Richard Duffin, he could encourage us to "think with our fingers" and to always maintain a balance between the conceptual and the computational.

Obviously, I'm enamored with Raoul Bott, and thrilled that he was the recipient of an honorary degree from McGill. However, my purpose here must be more focused. Specifically, in my mind, a few key points need elaboration:

- Who was the driving force behind getting McGill to award Bott an honorary degree in Mathematics forty years after he left McGill as an engineer? Clearly, Bott was deserving, but it takes a kindred spirit to overcome the inertia of a bureaucracy and Carl Herz was such a kindred spirit.
- In his convocation address, Bott vividly describes the pivotal moment at McGill when he decided to become a mathematician. However, how he was going to do it was not at all clear at the time. The wonderful and profound connections between topology and physics have been studied intensely in recent decades, but what is needed is a hint of the path from "engineering mathematics" to the mathematics Bott is known for[**Bo85**],[**JB**]. In retrospect, it almost seems that this path could have been more clear to

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[3]See "Reminiscences of Working with Raoul Bott" in [**STY**]

Gauss and Maxwell than to modern specialists. We'll soon see, the career of Hermann Weyl provides us with a perspective and some key insights.

- I'm fascinated with Bott's struggle to reconcile old and new world sensibilities. I saw this encoded in my interactions with him and in the convocation address. I use the word "encoded" in reference to the address, because his references to the uniquely Canadian iron ring have their roots in various Quebec City bridge disasters, Rudyard Kipling's poem "Sons of Martha," and associated Biblical references. These details are required to fully decode the message.

Carl Herz as a gateway to history

I can distinctly remember the day Carl Herz knocked me off my feet. At the time I was a graduate student in Electrical Engineering at McGill and he was a feisty and famous professor of Mathematics. We began to chat after some seminar in the EE department, and he asked me what I was doing for a thesis. He listened as I told him how I felt that the reformulation of Maxwell's equations in terms of differential forms was essential for the resolution of some key problems in computational electromagnetics. Specifically, most of the boundary value problems in low frequency electromagnetics amounted to Hodge theory on manifolds with boundary, with the periods of harmonic forms identified with the variables found in Kirhhoff's laws. Furthermore, I told him that the whole framework has a variational setting which can be discretized by appealing to "Whitney forms" in order to obtain a finite element discretization with desirable properties. To me it was all obvious if one read the papers written by Donald Spencer and his students in the 1950's and interfaced them with Whitney's "Geometric Integration Theory". In retrospect, this was a natural connection given the work of Jozef Dodziuk and Werner Muller's proof of the Ray-Singer conjecture, but it was not apparent at the time. Carl listened, started pacing back and forth and I was beginning to worry that he was going into a trance! I don't know what was going on in his mind, but I braced myself for what could come out of his mouth.

I think I was standing in stunned silence when Carl stopped and asked me if I ever read Maxwell. Sheepishly, I told him that I read a good deal of Maxwell and that everyone in my field swears by Maxwell. He then asked me if I knew what a periphractic number was. When I expressed my ignorance, he went on to point out that Betti's paper was written a decade after Maxwell's treatise, and that in Maxwell's treatise the first Betti number was called a "cyclomatic number" - a term introduced by Kirchhoff, and still used in graph theory. He went on to tell me that the second Betti number was called a "periphractic number"... I later found out that Maxwell borrowed the term from Listing[4] and that Listing was the person who coined the term topology. In one swoop Carl convinced me that Maxwell was often quoted but never read, and that if I wanted to get to the origin of these topological ideas the origin would be in some language other than English. Clearly, I was humbled — but I felt better when I looked up "periphractic" in the unabridged Oxford dictionary and found that Maxwell's treatise is the first and last use of the word in the English language![5]

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[4]See Breitenberger [**Br**] in James [**J**] for an article on Johann Benedikt Listing and his book.

[5]It is irresistable to point out the connection between Maxwell and Morse theory in this article about Bott. Listing[**L**] is credited as being the first to systematically obtain a cell decompositions

Besides being awed by Carl's encyclopedic knowledge, there are two big lessons I have leaned over the years and which were initiated by my encounter with Carl and other mathematicians from his generation who "read the masters." The first was that Maxwell had a profound experimental and theoretical knowledge, and that much of the inspiration for his theoretical work came from reading and corresponding with Germans (Gauss, Riemann, Kirchhoff, Clausius, Helmholtz, Listing, ...). Furthermore, it was the Germans who took Maxwell seriously when no one else did-from Helmholtz' student Hertz demonstrating radio waves, to Boltzman developing statistical mechanics, and to Einstein developing the logical physical conclusions of Maxwell's theory. Contrast this with the situation in England where Oliver Heaviside was considered a self-educated eccentric who died in poverty despite making brilliant contributions to Maxwell's theory and being awarded an honorary doctorate from Göttingen University in 1905. The other "Maxwellians" didn't make it into the limelight either.

The second big lesson I learned from my encounter with Carl is to never ignore the Institute for Advanced Study (IAS) in Princeton or the pround influence of its two first founding permanent members: Hermann Weyl and Albert Einstein. Carl was a student of Salomon Bochner and thrived on all the mathematics emanating from the IAS. In retrospect, the world seems quite small. It was Weyl who in 1948 invited Bott to the IAS, it was Weyl who earlier got de Rham, Kodaira, and Spencer to put Hodge theory on a rigorous footing, and it was Weyl [**W**],[**Y**] and his close colleague Einstein who were the true curators of the developments arising from Maxwell's theory. It turns out that the "Whitney forms" that I was so fond of have their origins in a 1952 paper of André Weil called "Sur les Theorems de de Rham." Clearly, every part of the novel mathematics I was using could be traced back to the IAS; even if my application of these ideas to computational electromagnetics was unforeseen. If ever I was in denial about details, I could check in with Donald Spencer's student, Robert Hermann, to verify facts[6]. If details were scarce in the literature, contemplating the influence of Hermann Weyl could help bring things into focus.

Enough said about my interactions with Carl Herz. To appreciate Carl Herz' contributions to harmonic analysis and other fields of mathematics, as well as the feisty character himself, through the eyes of his colleagues, the reader is referred to other sources [**D**], [**H**]. Needless to say, when I realized that Carl Herz was behind the effort to award Bott an honorary doctorate from McGill, it seemed like a big piece of the puzzle fell into place. He plays a central role on almost all correspondence with the university administration on the matter and in the end, he's the one who presented Bott for the degree in June of 1987.

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of 3-manifolds by tracking the change in topology as level sets cross a critical point. Maxwell [**M70**] then wrote a paper citing Cayley and Listing. In his treatise[**M**], Maxwell uses the rudiments of Morse theory with the fact that a harmonic function cannot achieve a maximum or minimum in the interior of a region in order to make topological deductions.

[6]The "Advanced Calculus" text Nickerson, Spencer, and Steenrod [**NSS**] was Princeton-inspired but was never published. However, it initiated a wave of differential-form based multivariable calculus texts in the 1960s. Although it is a very natural way to bring multivariable calculus to its roots in Physics, this wave of texts didn't catch on. Bott never wrote a text for such an undergraduate audience and so one can only hypothesize about how he would have integrated Kirchhoff's laws with Hodge theory and Maxwell's equations. In retrospect, it took a couple of decades to get things right and ultimately, the books that reached out to engineers and physicists most effectively were written by Bott's close colleagues [**BS**], [**F**].

The convocation presentation of Carl Herz

Mr. Chancellor,

I have the honour of presenting to you, in order that you may confer on him the degree of Doctor of Sciences, honoris causa, Professor Raoul Bott.

Raoul Bott was born in Hungary, but his university education up to the M. Eng. Was at McGill. He received his B. Eng. from McGill in 1945. After a short stint in the infantry, he continued his studies in electrical engineering at McGill. The immediate postwar period saw a great demand for mathematics teachers, and Bott taught calculus here while studying for his master's degree. In addition he took some courses from Professor Gilson, then Chair of the Department of Mathematics. Nevertheless, he remained a student of electrical engineering until he left McGill to go to Carnegie Tech for his doctorate.

Electrical engineering has a close affiliation with what might be viewed as an abstruse branch of mathematics, algebraic topology, Professor Bott's specialty. One has only to recall that "Betti numbers", the fundamental numerical invariants of topology, are named for an Italian electrical engineer, and one can read James Clerk Maxwell for profound insights into the subject. At an even more primitive level, circuit theory has always been a source of good problems for topologists. Bott's earliest work was rather algebraic. The Bott–Duffin Theorem (1949) on circuit synthesis was described by a reviewer thus: "This proof of the realizability of the driving point impedance without the use of transformers is one of the most interesting developments in network theory in recent years." It continues to be a much-cited result. This work came shortly after Bott had obtained a D.Sc. in mathematics.

After the doctorate, Raoul Bott went to the Institute for Advanced Study in Princeton. He was at the Institute during 1949–1951 and returned in 1955–1957. He joined the faculty of the University of Michigan in 1951 where he remained until 1959 when he was invited to his present academic home, Harvard, where he is William Caspar Graustein Professor of Mathematics.

Professor Bott's seminal contributions to mathematics are too extensive for me to do justice to them here. Most of his early ideas seem to have drawn their inspiration from the Calculus of Variations in its global version known as "Morse Theory". Bott applied Morse Theory in an unexpected and striking way. Over a long period he, together with his various collaborators, worked out the topology of Lie groups and symmetric spaces. One must mention the Bott Periodicity Theorem which brought some order to the chaos of homotopy theory. He went on to study fixed point theorems and their application to other branches of mathematics including differential equations. Most recently Bott has been working on applications of topology and geometry to the Yang-Mills equations in quantum field theory.

For his achievements, Bott was awarded the Veblen Prize of the American Mathematical Society in 1964.

In addition to his purely scientific accomplishment, Raoul Bott stimulates all those who are about him. He is one of the best and most exciting expositors of mathematics I have had the privilege to listen to.

Mr, Chancelor, McGill can take great pride in honoring this year, as it did last[7], another of its graduates who stand in the forefront of mathematics of the twentieth century.

The Eleventh Day of June, Nineteen Hundred and Eighty-seven

Carl Herz

Professor of Mathematics and Statistics.

Given my encounters with Prof. Herz, his correspondence with the McGill administration, and his encyclopedic breadth, it is clear that he played a central role in the case for the honorary degree. The masterful presentation of Carl Herz shows how a broad perspective can lead to a reorganization of knowledge that lets the likes of Paul Dirac and Eugene Wigner move from Engineering to Physics, and the likes of Raoul Bott, Solomon Lefschetz, John Milnor, and Donald Spencer move from Engineering to Mathematics. On the other hand, since Prof. Herz always enjoyed an argument (in the very best sense of the word!), I'll take the liberty to make a qualification and perhaps an eleboration.

The qualification I might add is that Enrico Betti was clearly not an electrical engineer but a mathematician. Indeed, Betti made contributions to both Elasticity theory and Electromagnetism, and Maxwell does indeed cite Betti's work in his treatise, but he was a mathematician. Betti, like the entire school of Italian Algebraic Geometry, was highly influenced by Riemann and topological ideas. However, the level of rigor in 19th century Italy was lax by modern standards and so his influence on current mathematical research may seem far removed. I revere Carl's respect for historical detail, and I'll refrain from calling his labeling Betti as an Electrical Engineer as a mistake. Rather I'd say Betti, like Gauss, Riemann, and Vito Volterra, had broad interests, and that Prof. Herz suppressed the pedant in himself and took some license in his interpretation of history.

The hidden hand of Hermann Weyl

The role of Hermann Weyl in getting mathematics off the ground in the earliest days of the IAS is now well documented [B]. What I find fascinating is the first and fateful encounter between Hermann Weyl and Raoul Bott. The encounter has a lot to do with interplay between electrical circuit theory, the early days of algebraic topology, and the perception of topology. The presentation of Carl Herz leaves out a lot of detail, much as a movie based on a book has to forgo a lot of detail. Given the encyclopedic knowledge of Carl Herz, it is tempting to speculate on what he could have put into a longer presentation.

Bott told the story of his first encounter with Hermann Weyl many times, emphasizing different aspects and different amounts of detail. See for example[Bo88]. I like the following rendition of the basic facts. During his grad student days as a

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[7]Raoul Bott, Jim Lambek and Louis Nirenberg all graduated from McGill in 1945 and Nirenberg was awarded an honorary doctorate from McGill in 1986.

student of Richard Duffin at Carnegie Tech, Bott played a large role in organizing the department colloquium. Being fluent in German, Hungarian, and Slovakian he would have an edge over other grad students in terms of "chatting up" foreign-born visitors. When Hermann Weyl visited, they were introduced, and Bott immediately began to tell Weyl of his thesis work[**BDu**]. There are interesting aspects of his thesis which predate both the Bott–Duffin Synthesis procedure and Wang algebras[**D59**]. One key aspect is the "impedance potential" and how it defines a generalized inverse of a matrix. Of course the Moore–Penrose axioms for a generalized inverse were only formulated in the 1950s and so Bott does not use the term.[8] Early on, he and Duffin called it a "constrained inverse"[**BDu**], and in expository talks Bott later described it in terms of orthogonal projections in a complex (i.e. Hodge theory). It turns out that his impedance potential is a determinant which is intimately related to what graph theorists call a "Matrix-tree formula"– a result that goes back to Kirchhoff and was used in Maxwell's treatise. The logarithmic derivative of the impedance potential with respect to branch impedances gives a generalized inverse. When Bott explained the formalism and associated results to Hermann Weyl, Weyl grasped that Bott-Duffin synthesis was indeed a contribution to network synthesis, but that the connection between Hodge theory and Kirchhoff's laws was not. He pointed Bott to some papers connecting Kirchhoff's laws to topology which he wrote in the early 1920s[9]. Needless to say, Bott was invited to the IAS, but Bott felt a bit deflated about the Hodge theoretic aspect and that Weyl saw it concretely in Kirchhoff's work.

To be fair to Bott, we have to ask why these papers of Hermann Weyl were so obscure. Gottingen was very closely tied to the technological aspects of Maxwell's theory, and so why were these two papers as obscure as Maxwell's periphractic numbers? What was the point Weyl was trying to make? To give some insight, a digression is in order.

In the Winter of 2005 I spent a month in the math department at the ETH in Zurich while on sabbatical. When I arrived, my host gave me a choice of offices: a huge office with a stunning view of Zurich belonging to a colleague on Sabbatical, or a very small empty office in the back of the building where "pure mathematicians hide their guests." I told my host that I wanted the freedom to "spread out," and that I felt more comfortable in the small back office. He was perplexed but obliged. It turns out that my cozy office was next to that of Beno Eckmann. On the centenary of Einstein's golden year, Zurich celebrated Eckmann as the last person in the city who had personal contact with Einstein! Since Hermann Weyl was the the head of the ETH mathematics department in the 1920s, I naturally wanted to pick Beno's brain for anecdotes. Not wanting to mess with his work habits, I planned to chat him up while he was a sitting target.

In the hallway outside our offices was a high-tech espresso machine and every morning Beno would take a break to sit and enjoy an espresso outside our offices. The first day, I "coincidentally" joined him and he related wonderful anecdotes from 1950-1955, after Hermann Weyl retired from the IAS study, resettled in Zurich,

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[8]See Chapter 2 and Appendix A of Ben-Israel and Greville's book [**BIG**] for an exposition that puts the Bott-Duffin constrained inverse in the context of generalized inverses, and for putting the "Moore" of "Moore-Penrose" in historical perspective.

[9]More precisely, Weyl's papers dealt with Kirchhoff's laws [**W23a**] and combinatorial topology [**W23b**].

and frequented the department. (I was sufficently impressed that when I returned to Boston, I contacted the editors of the Notices of the AMS and a year later the anecdotes appeared in print[**EWZ**]). The next day I resolved to ask Beno a question which I didn't think any living person could answer. Little did I know that he had written a paper on the subject [**E**] and had a definite opinion on every nuance I could ask him to elaborate on!

The conversation went something like this:

RK: Beno, there is something I really don't understand about Hermann Weyl.

BE: What is it?

RK: Well, in his collected works, there are are two papers about electrical circuit theory and topology dating from 1922/3. They are written in Spanish and published in an obscure Mexican mathematics journal. They are also the only papers he ever wrote in Spanish, the only papers published in a relatively obscure place, and just about the only expository papers he ever wrote on algebraic topology. It would seem that he didn't want his colleagues to read these papers.

BE: Exactly!

RK: What do you mean?

BE: Because topology was not respectable!

RK: Why was toplogy not respectable?

BE: Hilbert!

RK: Hilbert?

BE: Just look at his 23 problems from 1900. Do you see anything to do with combinatorial group theory or topology? No!

RK: Why?

BE: Poincaré![10]

RK: What did Hilbert think of Poincaré's work on toplogy?

BE: Poincaré would write a huge paper on Analysis Situs. Half of it would be completely wrong! So, he'd write another huge paper trying to correct the first, but it would be half wrong! And so he'd write a third paper, but it would be half wrong. And so on... deuxieme complement, troisieme, quatrieme, cinquieme,.. and in the end what did we get? Dubious results and conjectures! Hilbert didn't think this was mathematics!

RK: So why did Hermann Weyl write these papers?

BE: He wanted to take stock of the honest results and reorganize them using a more modern abstract algebraic approach. Emmy Noether and others were doing interesting things in algebra and he had a need to write these papers for himself. These papers also contain some new results like the signature of a 4-d manifold.

Beno went on to portray Hilbert as a bit of a reactionary figure, around which Hermann Weyl had to tip-toe. However, if Weyl wanted an opportunity to move things forward, it came in 1930 when Weyl succeeded Hilbert upon his retirement from Göttingen. Although he was only at the helm from 1930 until he fled the Nazis in 1933, during this very brief time German topology flowered in the hands of Emil Artin, Kurt Reidemeister, and others. According to Beno Eckmann, Weyl made a historic decision in 1930 which was highly controversial at the time, but ultimately vindicated: he appointed Heinz Hopf, a young researcher and relatively unseasoned, as his successor at ETH.

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[10]Sarkaria[**S**] has given a modern executive summary of Poincaré's work in Topology.

If Beno's historical perspective is taken superficially, there is a temptation to suspect there was some lasting disagreement between Weyl and Hilbert. However, one only needs to read Weyl's masterful summary of Hilbert's work [**W44**] to realize that both men held themselves to the highest standards. In a sense, every time Hilbert or Poincaré dug their heels in, Weyl found an opportunity to move mathematics forward. Algebraic topology may be one example and the continuum hypothesis may be another; perhaps the best example is the fact that Kurt Gödel was among the first four hires at the IAS, but unemployable in Europe.

What did Carl know? We can only speculate. I can only say that he is one of the many people who impressed upon me the importance of having a historical perspective when reconciling algebraic topology with its applications.

From History to Bott's reconciliation with it

The historical details discussed so far predate 1952. The next eight years would usher in the revolution in homotopy theory brought on by Serre's thesis, CW complexes which tie Morse theory to homotopy theory, Bott periodicity, generalized cohomology theory such as K-theory and Rene Thom's cobordism theory, and the reformulation of generalized cohomology theories in terms of spectra. Beno Eckmann pointed out to me that in the 1950s, those in Zurich who dismissed Hermann Weyl as an old man were favorably stunned by his summary of the work of Kunihiko Kodaira and Jean-Pierre Serre on the occasion of their being awarded the Fields Medal in 1954 [**W54**]. Nonetheless, contrasting Weyl's presentation of the work of the two Fields medalists, it is apparent that he was challenged by the homotopy theoretic world Bott had entered into, even if he did a lot to unleash the homotopy theoretic perspective. Enough said; it is time to leave threads of mathematical history and experience another view of history:

THE CONVOCATION ADDRESS OF RAOUL BOTT

Mr. Chancellor, Mr. Chairman of the Board– my dear fellow graduates:

Congratulations to you- class of '87! You look splendid! I think you wash more behind the ears than your American cousins at Harvard do.

It is nice to get a degree, isn't it? Of course you only had to work hard for four years or so to get yours, while it took me over forty years to get mine. And presumably you have paid for yours, while I am paying for mine at this moment by being here on this platform, making a fool of myself.

But, there is really nothing like one's first degree. And what I loved especially about my Bachelor of Engineering was that an iron ring (from a fallen bridge) came with it. I hope this tradition continues, so that at least you engineers, can contrive – as I did – to display it on every occasion. It is a marvelous way of starting a conversation and at the same time lets one know that you have "graduated." So my first admonition to you is: "Flaunt your degree in front of the whole world!" For a few weeks enjoy it to the hilt! The real world will rein you in soon enough.

Of course the people who enjoy your degree most are your parents. So by all means— here comes my second admonition — Get yourselves some children, in time for degree-harvesting when you are still in your forties! (That way you might also have time to repay the loans before you die.)

But let me tell you now a little bit about the good old days, just to keep some sort of historical perspective in a society, whose customs change at such a rate that the last forty years most probably represent two hundred uninflated ones.

First of all I must tell you that, beautiful as your Campus is today, it used to be even more so in 1941. There were lawns to stretch out on, there was even a tennis court by the Redpath library! There was so much space and such a fine line of proportion! And there were no skyscrapers! (On the other hand, the area around McGill was very rundown. And I see that our "greasy spoon" has now flowered into a pizza joint.)

Classes were small and some of my professors wore robes, as we are now, to teach in. They billowed and flowed delightfully with each step. These gowns were usually torn and completely covered with dust; still they added to the performance. I remember that later when I had my own calculus class to teach – the veterans had returned in huge numbers in the fall of 1945 and the Math. Department had pressed a lowly engineer into service to meet the demand – my dear friend and mentor, Prof. McLennan – the Socrates of our campus – lent me his well-weathered gown. "Try it in your class", he said, with a twinkle in his eye. Well, the class of course guffawed at first, but then actually settled down to work in a more businesslike manner than usual.

Possibly it was this ballet-like aspect of the lectures that kept me going to classes very diligently in the beginning. However, this epoch of my life came to an end in short order after one of my roommates in our boarding house on Durocher called me in for a serious talk. Elwood Henneman was his name and he continues as a dear friend and colleague at Harvard. Elwood, with the full authority of a first year medical student and a Harvard B.A. warned me of the danger of being addicted to classes. "Never become a slave to them," he declared; "do the bulk of your thinking on your own!" This point of view made immediate sense; and thereafter it was safest to look for me in the Music room. Usually in the company of my dear friend, Walter Odze, and much later on also with my wife to be.

How much this had to do with my falling grades I don't know, but in any case I did manage to always to "beat the Dean" as we used to say. Do you know what I mean? (But it sounds like good fun in any case, doesn't it?) Well, one of the endearing procedures of our Alma Mater at that time was that they didn't divulge our final grades until August – I think. Then suddenly your name was printed in the Gazette – if you passed, that is – and with an

asterisk if you flunked one course, etc. On the same day the names of all the passing students were listed on a billboard in linear order of merit. Those who had failed were not on the list, and at the bottom of this terrifying document came the signature of the dean! Hence the expression of beating the Dean if one got through.

But speaking of Deans and advice, let me tell you about one McGill Dean who in his own inimitable way gave me the best advice of my life. These were the war years and in '45 right after graduation, I joined up in the Canadian infantry and was being trained for combat in Japan. After three months in basic training the atomic bomb was dropped on Hiroshima and Nagasaki, the war ended abruptly and my fellow recruits and I thereby suddenly and miraculously reprieved – in this unbelievable and terrible manner.

Of course, the one great advantage of being in the army is that one has no career problems whatsoever! Hence the doubts I had about my vocation in engineering were completely submerged by my efforts to keep out of the Sergeant's hair. But when in October I found myself back in the Engineering Department, where they had very kindly let me return for a Master's Degree on – as you can imagine – very short notice, the old doubts flared up again and I was in a quandary about what to do.

It was sometime in '46 then that I presented myself at Dean Thompson's office and asked him whether he could see his way to putting me through medical school. (On the Jewish side of my family they always did say: "chutzpah he does not lack".) And Dean Thompson was quite encouraging at first. "We need scientifically trained doctors", he said. "But", he continued, "first tell me a little about yourself". It was at this point that our interview started to go sour. No, I never enjoyed Biology much. No I hated dissecting frogs. Alas, Botany bored me and I had little use for Chemistry! After this sorry litany, Dr. Thompson surveyed me and the situation for a while, pipe in hand, and lost in thought. "Is it maybe that you want to do good for humanity", he said at last. I hemmed and hawed in my seat, but before I had time to say anything he came out with: "Because they make the lousiest doctors!"

Well, that was it for me – and you must admit that it explains a lot of things, doesn't it? In any case, I got up and as I went to the door, I thought to myself: well you (explicative deleted) if that is how the land lies, then I will simply do what I like best: "I will become a Mathematician. Put that in your pipe and smoke it!"

I tell you all this only in part as a jest. I would also like it to be a word of encouragement to those of you who, degree in hand, still are not quite certain of your path. May you also be blessed with a counselor with such diagnostic skills and such a knack for putting you on the right course.

But my time is up! Still I cannot resist a serious word. Over my McGill days the War hung like an everpresent black cloud,

subtly affecting every aspect of our lives. For you in the nuclear age the cloud is, thank God, farther away, but potentially much, much darker. These things you will have to live with and somehow hope to conquer. But for this road I know of no better advice than my friend Elwood's - "Do your own thinking".

In our more immediate lives we are also beset today more than ever before, with show, with image, with jargon; and here again – to pick one's way through this quagmire, there is no better exhortation than: Be your own man; be your own woman. For then, I am confident, you will never confuse fashion with substance, heroes with the people who depict them on the tube, computers with people, Science with virtue, or wealth with happiness.

Yes, may God bless you and may you be joyfully and productively yourselves - but may you also be ultimately servants of a larger and an all-encompassing benign world view. And that is really no more than what I take to be the correct reading of Dr. Thompson's advice to me forty years ago. Only remember that the concerns of your generation must even transcend those off our human family. They must embrace every aspect of life itself on this deeply troubled, but magnificent and magical planet of ours.

Raoul Bott

On Iron Rings and other aspects of the Convocation Address

Bott's convocation address sets the stage for my fascination with his struggle to reconcile old and new world sensibilities. He was clearly the same age as my parents and like my parents, the disruption of his adolescence by Hitler, Stalin, and the events in the Europe of his youth was a traumatic experience and a profound education even if they didn't view it that way at the time[**BoD**]. His refugee experience was a stark contrast to the way kids grew up in North America during the decades after WWII. This was evident in his sense of humor and in the way he handled those who did not choose their words correctly. The testimonies of his students in this volume attest to this. The war years and the economic turmoil that preceded it left him with little tolerance for the dangerous comforts of self-preservation. The Engineer's iron ring along with its uniquely Canadian origin seem to frame some of his advice on responsibility and independent thinking. Reading the convocation speech, and recalling our interaction after the first class I audited at Harvard, it is apparent that Bott had a much more profound appreciation of, and respect for, the iron ring than did students he was addressing. One cannot do justice to the topic here[11], but it is useful to connect a few key ideas to events and sensibilities of other times.

Engineering is full of trade-offs, and the story of the iron ring is about the interface between technical trade-offs, ethics and ambition. One engineering trade-off is between the theoretical effort that goes into designing something without making a physical model, and the willingness to build prototypes and make mistakes. If one were to design a paper clip, one would make many prototypes in order to see "what works." On the other hand, if one were building a bridge, one would like to

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[11]These days one has to look for stainless steel if one wants to spot an "Iron Ring"—the iron of the early rings used to be eaten away by sweat and was soon replaced.

avoid disasters, and the development of a theoretical model with predictive properties is in order. In the case of bridge building, especially new designs, one is very cautious because public confidence is paramount. However, there have been many bridge disasters and contrary to what one might naively expect, they usually do not involve new designs! They usually involve refined designs which take into account that earlier designs were overly cautious, too costly, and less than ambitious. The temptations involved are quite universal and are not restricted to bridges; one can send up space shuttles routinely without an accident, but when one decides that the rules for launch are overly cautious in light of an opportunity to make some "State of the Union Address" spectacular, strange things can happen — just like when the detailed properties of O-rings were purposely ignored in the lead-up to the Challenger disaster. Similarly, the dismissing of foam impacts during launch as routine in the lead-up to the Columbia Shuttle disaster underlines the vigilance reqired to make complicated things work. These days we have "financial engineering" and computer models so predictive that there is a temptation to lose track of underlying assumptions and to consider the regulation of investors as unimaginative and cumbersome- here again, strange things can happen when ambition trumps regulation. Engineering disasters create teachable moments and they are very well documented when the stakes are high. In the case of bridges, the spectacular disasters have been studied and categorized, and scholarly books such as the one by Petroski[**P**] have been written.

Chapter three of Henry Petroski's book details the Quebec City bridge disaster(s) that lead to the iron ring worn by Canadian Engineers. It has a lot of detail on the New York based construction firm, the details of the bridge, the ignoring of warning signs and the 75 people killed in the first disaster. A key ingredient in this August 1907 disaster was an attempt to redesign the bridge during construction in order to ensure it broke a world record. There was a second disaster in 1916 during the construction of the redesigned bridge which killed 13 workers. In all, a total of 89 workers were killed in the construction of the bridge. The completion of the 1800 foot span of the Quebec City bridge in 1917 made it the largest cantilever brige in the world and vindicated the concept of the cantilever bridge for a mix of rail and automobile traffic. However, worldwide, no other major cantilever bridge was completed until the 1930s. To this day the Quebec City bridge has the longest span of any cantilever bridge – other bridge designs are used for longer spans. The original iron rings were made of the collapsed bridge's iron as a reminder of the stupidities engineers are capable of, and as a reminder of the engineer's responsibility to society – soon after the rings were made of stainless steel.

Rudyard Kipling was the recipient of the 1907 Nobel Prize in literature and lived in Bratleboro Vermont for a few years in the 1890s. It was in Vermont, between Quebec City and the home of the bridge's architect in New York, that he wrote "The Jungle Book." It was also in 1907, following the first Quebec City bridge disaster, that he wrote a poem called "The Sons of Martha".[12] It is inspired by the Gospel of Luke (10:38-42) and forms the basis of the original iron ring ceremony which Kipling was comissioned to write. The original Canadian ceremony was called "The Ritual of the Calling of an Engineer" and it was first performed in 1922. These days the original ceremony with its Biblical references is considered noninclusive.

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[12]See, for instance: "Kipling: A Selection of His Stories and Poems" by John Beecroft, (in two volumes), Doubleday 1956. In Vol. II, The Sons of Martha appears on p 451.

The iron ring ceremony, first performed in the United States in 1970, is centered around "The obligation of the engineer" which is devoid of Biblical references.[13] In Bott's commencement address he is clearly referring to the the original ceremony and I recommend a read through Kipling's poem to appreciate the iron ring as Bott would have been introduced to it.

This concludes my musings on a refugee as an engineering student in Montreal, his metamorposis into a topologist, and a new world success story who was ultimately awarded an honorary doctorate by his alma mater.

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[13]Kipling lived after "the days of wooden ships and iron men", and in the peak of the English Empire. Of the 89 workers killed in the two Quebec City bridge disasters, it appears that 33 were Mohawk steel workers form the Kahnawake reserve just outside of Montreal, creating 24 widows and numerous fatherless children. The Mohawk workers were well adapted to heights and were the "high tech" workers of their time. I have yet to see this aspect arise in the context of a modern iron ring cremony, or in the world view of Kipling's time.

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DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING, BOSTON UNIVERSITY, BOSTON, MA 02215

Current address: Department of Electrical and Computer Engineering, Boston University, Boston, MA 02215

E-mail address: prk@bu.edu