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★**Foundational essays on topological manifolds, smoothings, and triangulations.**

With notes by John Milnor and Michael Atiyah.
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From the days of Poincaré a major goal of topologists has been to understand the "naked" homeomorphism and its natural home—topological (TOP) manifolds. The lack of techniques to deal with these homeomorphisms forced Poincaré and early topologists to consider manifolds with a richer structure, namely infinitely differentiable (DIFF) or piecewise-linear (PL) manifolds, so that arguments of a combinatorial nature could be employed. The "triangulation conjecture" is that any TOP manifold is in fact a PL manifold, meaning that it is homeomorphic to a simplicial complex which is a so-called combinatorial manifold, i.e., the link of every simplex is PL-homeomorphic to a sphere. The Hauptvermutung, which was first put into print by Steinitz in 1907, conjectures that if a TOP manifold is homeomorphic to two combinatorial manifolds, then these combinatorial manifolds are PL-homeomorphic. The motivation for these conjectures was simply that the early understanding of homology groups required such combinatorial structures. After homology groups were shown actually to be topological invariants (around 1930), the Hauptvermutung and triangulation conjecture were advertised as being of independent interest. These conjectures were regarded as unapproachable until Milnor's discovery in 1956 of an "exotic" differentiable structure on S^7. Interest was rekindled in the Hauptvermutung and the study of topological manifolds was renewed in the 1960's. Important to the early progress in understanding the geometry of topological manifolds were, for example, the generalized Schoenflies theorem of M. Brown (which marked the advent of push-pull topology), the notion of a tangent microbundle of a TOP manifold developed by J. Milnor (which opened up the possibility of parallelling the delicate theories for classifying DIFF structures on PL manifolds in an understanding of the Hauptvermutung), and the development of engulfing in studying topological manifolds, for instance, M. H. A. Newman's proof of the topological Poincaré conjecture. Then, in the very exciting year 1968, A. Černavskiĭ developed a meshing technique to show that the homeomorphism group of a compact TOP manifold is locally contractible, and the first author developed a torus unfurling technique to reduce the annulus conjecture to a question concerning the Hauptvermutung for tori. It was the second author's idea that even though the nonsimply connected surgery theory developed by C. T. C. Wall showed that an obstruction in $H^3(T^n; Z_2)$ exists to finding a PL-homeomorphism between a PL manifold homeomorphic to the n-torus T^n and T^n itself, this obstruction might vanish under liftings to 2^n-fold covers: this would be enough for Kirby to prove the annulus conjecture. This was shown in late 1968 to be indeed the case by Wall, W. C. Hsiang and J. Shaneson, and by A. Casson. Kirby's crucial idea—the so called "main diagram"—was then extended in joint work of the authors not only to disprove the triangulation conjecture and Hauptvermutung but also to extend geometric techniques, previously available only for DIFF or PL manifolds, to topological manifolds—namely the techniques of transversality, handle decompositions and simple homotopy theory. The tightly kept secrets of the naked homeomorphism had been revealed!

The five essays under review, which have been in existence in various forms since 1970, represent the first full publication of these new results. Essay I proves two basic results: any two CAT (= DIFF or PL) structures on a TOP manifold which are concordant are isotopic (the concordance implies isotopy theorem); and a CAT structure θ on $M \times R^n$ induces a CAT structure σ on M with $\sigma \times \mathbf{R}^n$ concordant to θ (the product structure theorem). These results are proved in a strong relative form which allows a reduction to a coordinate chart which admits a handle decomposition and then by induction it is enough to consider the special case when M is a handle. This "handle straightening" is the keystone of these essays. In Essay III the geometric techniques listed above are extended to topological manifolds using the known geometry of PL and DIFF manifolds and the results from Essay I. Essay IV treats the classification of CAT manifold structures on a TOP manifold in terms of a stable reduction of the TOP tangent bundle to a CAT bundle, thus yielding a homotopy classification of CAT manifold structures and information about the stability of $\mathrm{TOP}_m/\mathrm{CAT}_m$. The calculation of the homotopy groups of TOP/CAT is delayed until Essay V. Essays II and V should be grouped together. A competing approach to the classification theorem is to exploit ideas that first arose in the Smale-Hirsch classification of immersions. These ideas dictate proving a theorem that sliced concordance implies isotopy and then a nonstable classification theorem for sliced (parameterized) families of CAT structures. Consequently, nonstable information about the homotopy groups of $\mathrm{TOP}_m/\mathrm{CAT}_m$ is obtained. This approach was put forth by C. Morlet through his observation that the isomorphisms $\pi_{m+k+1}(\mathrm{TOP}_m/\mathrm{CAT}_m) \cong \pi_k(\mathrm{Aut}_{\mathrm{CAT}}(B^m \mathrm{rel}\, \partial B^m))$ follow naturally from a sliced theory. Essays II and V present such an approach; techniques are presented which are worth learning for their many proven and future applications. Essay II proves, in a very elegant fashion, that a CAT submersion is in fact a bundle projection. Using the CAT isotopy extension theorem, this is straightforward when the "fiber" is compact; but it requires an ingenious proof when the "fiber" is noncompact. This bundle theorem is then extensively used in Essay V to prove the sliced classification theorem, again by an extremely elegant argument. This classification theorem is then applied to calculate the homotopy groups of $\mathrm{TOP}_m/\mathrm{CAT}_m$, to prove a strong stability theorem for TOP_m, and to recover the above-mentioned isomorphisms.

An all-out effort was made to make this book self-contained. The authors present the material both expertly and well. There is a reluctance of the authors to abandon any of several competing approaches, which to some might be a fault; but to the reviewer, it is refreshing. Not only are the tasks at hand dealt with, but a framework is developed which has been and perhaps still will prove to be useful in handling other manifold questions of a topological nature. This self-containment is partially enhanced by several appendices to the essays. For instance, Casson's beautifully simple construction of a fake torus starting with the Poincaré homology 3-sphere is given in Appendix B of Essay IV. In Appendix A of Essay V a mild extension of the immersion-theoretic method is given which is a chart-by-chart procedure useful for proving many classification theorems which rely on so-called homotopy micro-lifting properties singled out by Gromov. Also, a summary of the necessary results from the surgery theory of Wall is given in Appendix B of Essay V along with the important surgical classification of homotopy tori originally due to Casson, Hsiang and Shaneson, and to Wall.

This book is indispensable for anyone interested in topological manifolds. The neophyte should keep in mind what is set out to be proved; he should follow a line of least resistance, keeping alternative approaches and enlightening generalizations for later readings. Before a framework can be developed, a foundation must be laid. The book is demanding, but self-contained, well written, and its readers will be rewarded with a full and open understanding.

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