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MR2375818 (2009c:57042)

Levine, Jerome(1-BRND)

Concordance of boundary links. (English summary)

J. Knot Theory Ramifications 16 (2007), no. 9, 1111--1120.
57Q45 (57M25 57Q60)

Citations
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This paper proves the concordance invariance of certain Atiyah-Patodi-Singer ρ-invariants of links. The importance of this result can be understood in the context of on-going efforts to understand links up to concordance.

Roughly speaking, high-dimensional links with m components can be classified by invoking the Cappell-Shaneson homology surgery theory with the Vogel homology localization of the wedge of m circles as the target space, due to work of J.-Y. Le Dimet [Mém. Soc. Math. France (N.S.) No. 32 (1988), {\rm ii}+92 pp.; MR0971415 (90e:57046)]. Nonetheless, the structure of link concordance classes is still largely unknown, because of the lack of understanding of the Vogel homology localization and related algebraic and homotopy theoretic problems.

On the other hand, if one thinks of smaller classes of links (and appropriate notions of concordance), classifications are more accessible. The most important case is boundary concordance of boundary links, which were classified by S. E. Cappell and J. L. Shaneson [Comment. Math. Helv. **55** (1980), no. 1, 20--49; MR0569244 (81j:57011)] in terms of homology surgery obstructions, by K. H. Ko [Trans. Amer. Math. Soc. **299** (1987), no. 2, 657--681; MR0869227 (88h:57018)] and W. Mio [Math. Proc. Cambridge Philos. Soc. **101** (1987), no. 2, 259--266; MR0870597 (88e:57023)] in terms of Seifert matrices, and by J. Duval [C. R. Acad. Sci. Paris Sér. I Math. **299** (1984), no. 18, 935--938; MR0774672 (86c:57020)] in terms of Blanchfield forms. In 2001, D. Sheiham gave a full computation of the structure of boundary concordance group of boundary links [Mem. Amer. Math. Soc. **165** (2003), no. 784, x+110 pp.; MR1997849 (2005f:57038)].

It is known that the concordance classification of links is not reduced to the boundary concordance classification of boundary links; T. D. Cochran and K. E. Orr proved that not all links are concordant to boundary links [Ann. of Math. (2) **138** (1993), no. 3, 519--554; MR1247992 (95c:57042)]. But, for boundary links, whether concordance and boundary concordance are the same still remains as one of the most important open problems. In this regard, one is naturally led to investigate concordance \text{(non-)}invariance of boundary concordance invariants.

In the paper under review, the author considers the boundary link concordance invariants defined in his earlier work [Comment. Math. Helv. **69** (1994), no. 1, 82--119; MR1259607 (95a:57009)]. To avoid technical complications, he states results for m-component boundary links $L\subset S^n$ endowed with a choice of a homomorphism $\pi_1(S^n-L) \to F$ sending meridians to generators, where F denotes the free group of rank m. Such a link is called an F-link. Suppose

$n=2q+1$. Let $R_k(F)$ be the space of k-dimensional unitary representations of F, which is a real algebraic variety. For a given F-link L, the author defines a function $\rho_L\colon R_k(F) \to \Bbb{R}$, which is the Atiyah-Patodi-Singer ρ-invariant of the surgery manifold M_L of L. He also defines a subvariety $D_L\subset R_k(F)$ of representations θ such that $H_q(M_L;\Bbb{C}^k)$ with coefficients given by θ is nontrivial.

The main result is the following (ordinary) concordance invariance of ρ_L: If L_0 and L_1 are concordant (or give concordant disk links), then ρ_{L_0} and ρ_{L_1} are equal on $R_k(F)-(D_{L_0} \cup D_{L_1})$. For a boundary slice link L, $\rho_L=0$ on $R_k(F)-D_L$.

In [op. cit.], Sheiham proved that signature invariants obtained from certain quiver representations determine the boundary concordance classes modulo torsion. From the result of the paper under review, it follows that if all the Sheiham signatures are captured by the ρ-invariants, then boundary concordance is equal to ordinary concordance modulo torsion. Including this, some interesting open problems are listed in the last section of this paper.

Reviewed by Jae Choon Cha

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MR2286030 (2007k:57006)

Cimasoni, David(1-CA)

Slicing Bing doubles. (English summary)

Algebr. Geom. Topol. 6 (2006), 2395--2415.
57M25 (57M27)

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A knot $K\subset S^3$ is called slice if it bounds a smoothly embedded disk $D\subset D^4$. Furthermore, a link is called slice if the components bound disjointly embedded disks in D^4. One of the most interesting questions in the study of slice links is the question of how certain topological constructions affect the sliceness of a link. In this paper the author studies the effect of Bing doubling on the sliceness of a knot. The Bing double $B(K)$ of a knot K is the result of wrapping the trivial link with two components in a certain nontrivial way around K. Given a slice knot K it is well-known that the Bing double $B(K)$ is a slice link. It is a natural question to ask whether the converse holds, i.e. whether $B(K)$ slice implies that K is slice. Some evidence for an affirmative answer had been given independently by P. Teichner and S. Harvey in their study of the L^2-von Neumann ρ-invariant of $B(K)$.

In the paper under review the author first shows that the Bing double of a knot K is a boundary link, i.e. the components bound disjoint Seifert surfaces in S^3. In the case of a boundary link there is a stronger notion of sliceness: a boundary link is called boundary slice if it is slice and if the union of the Seifert surfaces and the slice disks bound disjoint embedded 3-manifolds in D^4. It is not known whether a slice boundary link is necessarily boundary slice.

The main theorem of this paper is the following:

Theorem. If the Bing double $B(K)$ is boundary slice, then the knot K is algebraically slice.

Here a knot K is called algebraically slice if its Seifert matrix has a metabolizer.

This theorem is remarkable as it is the first theorem of its kind to completely capture the information on the algebraic concordance class of K. The proof of this theorem relies heavily on D. Sheiham's study of boundary slice concordance [Mem. Amer. Math. Soc. **165** (2003), no. 784, x+110 pp.; MR1997849 (2005f:57038)].

Cimasoni's theorem has since been strengthened by J. C. Cha, C. Livingston and D. Ruberman [``Algebraic and Heegaard-Floer invariants of knots with slice Bing doubles'', Math. Proc. Cambridge Philos. Soc., to appear]; in fact, they showed that in the theorem it is enough to assume that $B(K)$ is slice.

Finally, in the last section the author gives further evidence to the conjecture that a knot is slice if and only if its Bing double is slice by studying the Rasmussen-Beliakova-Wehrli invariant of links.

Reviewed by Stefan K. Friedl

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MR2284050 (2008g:57026)

Ranicki, Andrew(4-EDIN-SM); Sheiham, Desmond

Blanchfield and Seifert algebra in high-dimensional boundary link theory. I. Algebraic K-theory. (English summary)

Geom. Topol. 10 (2006), 1761--1853 (electronic).
57Q45 (19D35 20E05)

Citations
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Let L be a boundary link of m copies of S^n in S^{n+2}. The free cover $\tilde E$ of the exterior E of the link is tesselated by copies of E cut open along a collection V of disjoint Seifert surfaces of the components of L [cf. J. A. Hillman, *Algebraic invariants of links*, World Sci. Publ., River Edge, NJ, 2002; MR1932169 (2003k:57014); N. Sato, Trans. Amer. Math. Soc. **264** (1981), no. 2, 499--505; MR0603777 (82j:57020)]. In particular, the chain complex $C_*(\tilde E)$ is a quotient of ${C_*(S^{n+2}\sbs V)[F_m]}$ with kernel isomorphic to $C_*(V)[F_m]$. Here F_m is the free group on $t_1,\dots,t_m$, and the inclusion of the kernel is given by $(f_1^+-t_1f_1^-,\dots,f_m^+ -t_mf_m^-)$, where $f_i^\pm$ are induced by the two pushoffs of the ith component of V into $S^{n+2}\sbs V$. The authors show that not only $C_*(\tilde E)$, but every chain complex of $A[F_m]$-modules, where A is an associative ring with 1, admits a presentation of this type (Theorem A). The case $m=1$ was proved by F. Waldhausen [in *Algebraic K-theory, II: ``Classical'' algebraic K-theory and connections with arithmetic (Proc. Conf., Battelle Memorial Inst., Seattle, Wash., 1972)*, 155--179, Lecture Notes in Math., 342, Springer, Berlin, (1973); MR0370576 (51 #6803)], and the authors note that the general case can also be deduced from his results.

This ``algebraic transversality'' is used to obtain decompositions of the algebraic K-groups $K_i(A[F_m])$ (Theorem D(i)) and of the negative K-groups of the Cohn localization $\Sigma^{-1}A[F_m]$ (Theorem E). The latter decomposition extends to K_1 and, if $\Sigma^{-1}A[F_m]$ is stably flat, also to the higher K-groups. The stable flatness condition holds when A is a PID and is not known to fail in general. In the case $m=1$ these results specialize to the Bass-Heller-Swan decomposition of $K_1(A[t^{\pm 1}])$ and its analogue for the higher K-groups, and to a splitting theorem of the first author [*High-dimensional knot theory*, Springer, New York, 1998; MR1713074 (2000i:57044)]. The case $A=\Bbb Z$, in

which $K_i(\Bbb Z[F_m])$ has been long known to be the direct sum of $K_i(\Bbb Z)$ and m copies of $K_{i-1}(\Bbb Z)$, is discussed in Remark 5.17 at the very end of the paper.

The new decompositions involve the algebraic K-groups of categories of Seifert and Blanchfield modules, which are abstract versions of the Seifert and Blanchfield modules of m-component boundary links. A ``Seifert module'' is a triple consisting of an A-module P, its endomorphism, and a system of idempotents expressing P as an m-fold direct sum. A Blanchfield module (previously called an F_m-link module by the second author [in *Non-commutative localization in algebra and topology*, 143--219, Cambridge Univ. Press, Cambridge, 2006; MR2222485 (2007h:57035)]) is an $A[F_m]$-module M such that the A-module homomorphism $M\oplus\dots\oplus M\to M$ defined by $(a_1,\dots,a_m)\mapsto (t_1-1)a_1+\dots+(t_m-1)a_m$ is an isomorphism. In the present paper, the authors observe that M is Blanchfield if and only if $\roman{Tor}^{A[F_m]}_*(A,M)=0$. The ``algebraic transversality'' is used to establish a long exact sequence relating the algebraic K-groups of the categories of Blanchfield and Seifert modules (Theorem D(ii)), which is new even in the knot case $m=1$.

Reviewed by Sergey A. Melikhov

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Sheiham, Desmond

Universal localization of triangular matrix rings. (English summary)

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Let R be a triangular matrix ring $\left(\smallmatrix A&M\\0&B\endsmallmatrix \right)$ where A and B are associative rings with 1 and M is an (A,B)-bimodule. Let P and Q be the finitely generated projective modules $\left(\smallmatrix A\\ 0\endsmallmatrix\right)$ and $\left(\smallmatrix M\\ B \endsmallmatrix\right)$, respectively. Given a morphism $\sigma\colon P \rightarrow Q$, this paper gives a description of the ``universal localization'' $R \rightarrow \sigma^{-1}R$ which is universal with respect to the property that $1\times\sigma\colon\sigma^{-1}R \otimes_RP\rightarrow\sigma^{-1}R \otimes_RQ$ is an isomorphism. A construction of the ``universal localization'' of an R-module is also given.

Reviewed by Mary H. Wright

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MR2222649 (2007f:55002)

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Non-commutative localization in algebra and topology. Proceedings of the workshop held in Edinburgh, April 29--30, 2002. Edited by Andrew Ranicki. London Mathematical Society Lecture Note Series, 330. *Cambridge University Press, Cambridge,* 2006. xiv+313 pp. ISBN: 978-0-521-68160-5; 0-521-68160-X
55-06 (16-06 55P60)

Contents: Andrew Ranicki, Dedicated to the memory of Desmond Sheiham (13th November 1974--25th March 2005) (vii--viii) MR2222477; Andrew Ranicki, Preface (ix) MR2222650; John A. Beachy, On flatness and the Ore condition (1--4) MR2222478; P. M. Cohn, Localization in general rings, a historical survey (5--23) MR2222479; William G. Dwyer, Noncommutative localization in homotopy theory (24--39) MR2222480; Peter A. Linnell, Noncommutative localization in group rings (40--59) MR2222481; Amnon Neeman, A non-commutative generalisation of Thomason's localisation theorem (60--80) MR2222482; Andrew Ranicki, Noncommutative localization in topology (81--102) MR2222483; Holger Reich, L^2-Betti numbers, isomorphism conjectures and noncommutative localization (103--142) MR2222484; Desmond Sheiham, Invariants of boundary link cobordism. II. The Blanchfield-Duval form (143--219) MR2222485; Zoran Škoda, Noncommutative localization in noncommutative geometry (220--313) MR2222486.

{Most of the papers are being reviewed individually.}

MR2222485 (2007h:57035)

Citations
From References: 3
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Sheiham, Desmond
Invariants of boundary link cobordism. II. The Blanchfield-Duval form. (English summary)
Non-commutative localization in algebra and topology, 143--219, London Math. Soc. Lecture Note Ser., 330, *Cambridge Univ. Press, Cambridge,* 2006.
57Q45 (19G12)

The Blanchfield-Duval form is used to define a complete set of invariants of boundary links ${S^n\times\{1,\dots,m\}}\hookrightarrow S^{n+2}$, $n\ge 2$, endowed with a choice of disjoint Seifert surfaces, up to concordance extending to disjoint cobordisms of the Seifert surfaces. Here $n=2q-1$ since in the even-dimensional case this equivalence relation is well known to have just one equivalence class.

In Part I [Mem. Amer. Math. Soc. **165** (2003), no. 784, x+110 pp.; MR1997849

(2005f:57038)], the author already defined such a complete set of invariants in terms of Seifert forms, which is ``convenient in explicit computations''. The objective of the present Part II is to redefine these invariants in ``more intrinsic'' terms, which are also ``more amenable to generalization from boundary links to arbitrary links''.

The choice of the cobordism class of Seifert surfaces is equivalent, by the Pontryagin construction, to a choice of a homomorphism $h\colon \pi_1(X)\to F_m$ sending meridians to generators, where X is the link exterior and F_m is the free group on m generators. The Poincaré duality in the cover X_ω of X corresponding to the kernel $\gamma_\omega\pi_1(X)$ of h gives rise to a version of the Blanchfield pairing.

The Blanchfield pairing is best understood and easiest to describe in the case of the free abelian cover $\tilde X$ [cf. J. A. Hillman, *Algebraic invariants of links*, World Sci. Publ., River Edge, NJ, 2002; MR1932169 (2003k:57014)]. Given two q-cycles x, y in $\tilde X$ such that $\alpha x=\partial u$ and $\beta y=\partial v$ for some nonzero $\alpha,\beta\in\Bbb Z[\Bbb Z^m]=\Bbb Z[t_1,\dots,t_m]$ and some $(q+1)$-chains u, v, the Blanchfield pairing $B(x,y)=\alpha^{-1} \sum_{g\in\Bbb Z^m} (u\cdot gy)g$ is a rational function in $t_1,\dots,t_m$, well-defined up to addition of finite polynomials.

The Blanchfield-Duval pairing on $H_q(X_\omega)$ is a noncommutative analogue of this construction. The polynomial ring $\Bbb Z[\Bbb Z^m]$ is replaced by $\Bbb Z[F_m]$, and its field of fractions $\Bbb Q(t_1,\dots,t_m)$ by the Cohn localization of $\Bbb Z[F_m]$, defined by formally adjoining inverses to all square matrices with entries from $\Bbb Z[F_m]$ that become invertible under the augmentation $\Bbb Z[F_m]\to\Bbb Z$. It was shown by Farber and Vogel [cf. M. S. Farber, Math. Ann. **293** (1992), no. 3, 543--568; MR1170526 (94m:68137)] that the Cohn localization of $\Bbb Q[F_m]$ can be identified with the subring of $\Bbb Q\langle\langle t_1,\dots,t_m\rangle\rangle$ consisting of those noncommutative power series that have the form $\sum f(T_{i_r}\dots T_{i_1} v) t_{i_1}\dots t_{i_r}$ for some vector $v\in V$, linear function $f\in V^*$ and transformations $T_1,\dots,T_m\in V\otimes V^*$ on a finite-dimensional $\Bbb Q$-vector space V.

In Section 2, the author discusses Blanchfield-Duval forms in more detail and defines their Witt groups. He works out complete invariants of these groups in Section 3. Sections 4--6 identify these invariants with those constructed in Part I.

Seifert forms have already been related to Blanchfield-Duval forms in the present context by Farber [op. cit.] and by T. D. Cochran and K. E. Orr [Topology **33** (1994), no. 3, 397--427; MR1286923 (95f:57041)]; see also [S. Garoufalidis and J. P. Levine, J. Knot Theory Ramifications **11** (2002), no. 3, 283--293; MR1905685 (2003f:57012)]. In the author's own words: ``Although the present paper is logically independent of [Farber's] work, we take up a number of his ideas in Sections 4 and 5, providing a systematic treatment in the language of Hermitian categories. Whereas Farber takes coefficients in a field or in $\Bbb Z$, in these sections we allow the coefficients to lie in an arbitrary associative ring A.''

Let us note that the relation of the Blanchfield-Duval form with the Seifert form is parallel to the connection between the Alexander polynomial and the Conway polynomial (as defined by Kauffman in terms of the Seifert form, see [D. Cimasoni, Comment. Math. Helv. **79** (2004), no. 1, 124--146; MR2031702 (2005d:57005)] along with the reviewer's preprint [``Colored finite type invariants and a multi-

variable analogue of the Conway polynomial'', preprint, \url{arxiv.org/abs/math.GT/0312007}] for the multi-variable case), as well as to the connection [T. D. Cochran, Comment. Math. Helv. **60** (1985), no. 2, 291--311; MR0800009 (87f:57021)] between the Kojima-Yamasaki η-function and Cochran's derived invariants (as defined in terms of a Seifert form [see P. M. Gilmer and C. Livingston, Proc. Edinburgh Math. Soc. (2) **34** (1991), no. 3, 455--462; MR1131964 (93a:57024)]).

It is terribly sad to learn that Desmond Sheiham passed away, only a few years after completing his PhD---and 50 years after Richard Blanchfield suffered a similar fate. The investigations of the present paper are continued in [A. A. Ranicki and D. Sheiham, Geom. Topol. **10** (2006), 1761--1853 (electronic); MR2284050].

{For the entire collection see MR2222649 (2007f:55002).}

Reviewed by Sergey A. Melikhov

MR2222477

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Dedicated to the memory of Desmond Sheiham (13th November 1974--25th March 2005). *Non-commutative localization in algebra and topology,* vii--viii, London Math. Soc. Lecture Note Ser., 330, *Cambridge Univ. Press, Cambridge,* 2006.

01A70

{This item will not be reviewed individually.}

{For the entire collection see MR2222649.}

MR2016661 (2004i:16014)

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Sheiham, Desmond(1-CAR)

Whitehead groups of localizations and the endomorphism class group. (English summary)

J. Algebra 270 (2003), no. 1, 261--280.

16E20 (19B99)

Let A and B be associative rings. A ring homomorphism $\epsilon\colon B\to A$ is said to be local in case a square matrix α with entries in B is invertible if $\epsilon (\alpha)$ is invertible. The homomorphism ϵ is said to be an augmentation if there is a ring homomorphism $j\colon A\to B$ such that $\epsilon j=1_A$. Every augmentation $\epsilon\colon B\to A$ induces a local augmentation $\epsilon_{\Sigma}\colon \Sigma ^{-1}B\to A $, where Σ is the set of all square matrices α over B such that $\epsilon (\alpha)$ is invertible, and $\Sigma ^{-1}B$ is the universal localization of B with respect to Σ.

The main result of the paper is as follows. Let $\epsilon\colon B\to A$ be a local augmentation. Then $K_1(B)\cong K_1(A)\oplus \epsilon^{-1}(1)/C$, where C is the subgroup of $\epsilon^{-1}(1)$ generated by the elements of the form $(1+ab)(1+ba)^{-1}$, where $a,b\in B$ and $\epsilon (ab)=\epsilon (ba)=0$. This result is applied to the computation of the Whitehead groups of (twisted) formal power series rings and the augmentation localizations of group rings and polynomial rings. Using a result of A. Ranicki [*High-dimensional knot theory*, Springer, New York, 1998; MR1713074 (2000i:57044) (Proposition 10.21)], the author obtains from his main

result an isomorphism $\widetilde{\text{End}}_0(A)\cong \epsilon_{\Sigma}^{-1}(1)/C$, where $\epsilon _{\Sigma}\colon \Sigma ^{-1}A[x]\to A$ is the local augmentation induced by the augmentation $\epsilon \colon A[x]\to A$, $x\mapsto 0$, for any associative ring A.

Reviewed by Pere Ara

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MR1997849 (2005f:57038)

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Invariants of boundary link cobordism. (English summary)
Mem. Amer. Math. Soc. 165 (2003), no. 784, x+110 pp.
57Q45 (19G12 57Q60)

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An algorithm to decide whether two given n-knots $S^n\subset S^{n+2}$, $n>1$, are concordant was given by J. Levine [Invent. Math. **8** (1969), 98--110; addendum, ibid., 355; MR0253348 (40 #6563)]. The very basic ingredient was the existence of a Seifert surface, i.e., a submanifold $V^{n+1}\subset S^{n+2}$ with $\partial V=S^n$. (Here all submanifolds can be assumed to be PL locally flat; originally they were smooth, but with possibly exotic smooth structures on S^n and $S^n\times I$.) In an attempt to generalize the Seifert surface approach to the m-component case, $m>1$, one is often content to obtain results for boundary n-links, whose components bound disjoint Seifert surfaces. Still, the generalization is usually not easy, amounting to a transition from $\bold Z$ to the nonabelian free

group F_m. Indeed, collections of disjoint Seifert surfaces for an n-link $\coprod S^n\subset S^{n+2}$ up to disjoint cobordisms rel ∂ correspond via the Pontryagin construction to epimorphisms $\pi_1(S^{n+2}\coprod S^n)\to F_m$, taking meridians onto conjugates of the elements of some fixed basis $\{x_1, \dots,x_m\}$.

The main result of this memoir is an algorithm to decide whether two given boundary n-links, $n>1$, with given collections of disjoint Seifert surfaces can be joined by a concordance extending to disjoint cobordisms of the Seifert surfaces. The author's approach is based on the Seifert form and extends M. A. Kervaire's treatment of the one-component case [in *Manifolds--Amsterdam 1970 (Proc. Nuffic Summer School)*, 83--105, Lecture Notes in Math., 197, Springer, Berlin, 1971; MR0283786 (44 #1016)], which may be a valuable complement to the memoir for the geometrically-minded reader. In his recent preprint [D. Sheiham, ``Invariants of boundary link cobordism. II. The Blanchfield-Duval form'', preprint, arxiv.org/abs/math.AT /0404229] the author gives an alternative classification in terms of the Blanchfield-Duval form. The difference between the two forms is akin to that between the Conway and Alexander polynomials in classical knot theory.

For $n>1$ the equivalence classes of pairs (m-component boundary n-link, collection of disjoint Seifert surfaces) up to concordance, extending to disjoint cobordisms between the Seifert surfaces, form an abelian group C^n_m (addition is given by connected sum of representatives along any collection of meridians mapping onto $\{x_1,\dots,x_m\}$, and inverse by reflection of both domain and range). S. Cappell and J. Shaneson showed that $C^{2q}_m=0$ for all q and m, and Levine [op. cit.] proved that $$C^{2q-1}_1\simeq\bigoplus_{i=1}^ \infty(\bold Z\oplus\bold Z/2\oplus\bold Z/4)$$ for each $q>1$. As a corollary to his algorithmic classification, Sheiham establishes that $$C^{2q-1}_m\simeq \bigoplus_{i=1}^\infty (\bold Z\oplus\bold Z/2\oplus\bold Z/4\oplus\bold Z/8)$$ for each $q>1$ and $m>1$.

His starting point is a description of C^{2q-1}_m due to K. Ko and W. Mio. They showed that C^{2q-1}_m is isomorphic for each $q>2$ to the Witt group of bilinear forms λ on finitely generated free abelian groups $M=\bigoplus_{i=1}^m M_i$ such that $\phi\coloneq \lambda+(-1)^q\lambda^t$ is non-singular and satisfies $\phi(M_i,M_j)=0$ for $i\ne j$. In particular, $C^n_m\simeq C^{n+4}_m$ for $n\ne 1,3$; as for C^3_m, it is an index 2^m subgroup in C^7_m. Geometrically, ϕ is the intersection form on the union of the Seifert surfaces, and the Seifert form $\lambda(x,y)$ is the linking number of x with the $+$-pushoff of y.

Ordinary Witt groups are defined for forms with some kind of symmetry, yet λ need not be symmetric. But there is some additional symmetry in ϕ: if M is regarded as a $\bold Z[s]$-module, with $s\colon M\to M$ acting by $\phi(sx,\cdot)=\lambda(x,\cdot)$, ϕ becomes $(-1)^q$-Hermitian (i.e., $\phi(x,y)=(-1)^q\overline{\phi(y,x)}$\,) with respect to the involution $\overline s=1-s$ on $\bold Z[s]$. To take into account the block structure of M for $m>1$, the idempotents $\pi_i\colon M\to M_i\to M$ are assembled formally with s into the noncommutative ring $$P_m\coloneq\bold Z\langle s,\pi_1,\dots,\pi_m\rangle/ (\pi_i^2-\pi_i,\pi_i\pi_j,\pi_1+\dots+\pi_m-1).$$ Thus C^{2q-1}_m, $q>2$, becomes the Witt group $W^{(-1)^q}(P_m\text{-}\bold Z)$ of $(-1)^q$-Hermitian forms on modules over P_m, which are finitely generated projective over $\bold Z$. The ring P_m was introduced by M. S. Farber [Math. Ann. **293** (1992), no. 3,

543--568; MR1170526 (94m:68137)], and turns out to coincide with the path ring of the complete quiver on m vertices (with m^2 arrows). This interpretation is used to show that there are only finitely many finite-dimensional simple representations of P_m over $\bold Q$.

Tensoring by $\bold Q$ leads to an injection $$W^\pm(P_m\text{-}\bold Z)\to W^\pm(Q_m\text{-}\bold Q),$$ where Q_m denotes $P_m\otimes\bold Q$. The Hermitian Jordan-Hölder theorem identifies the Witt class of $(M\otimes\bold Q,\phi)$ with the direct sum of the Witt classes of its simple subfactors (N_i,ϕ_i). Each N_i is simple as a Q_m-module, so each ϕ_i is either non-singular or metabolic. Hence the Hermitian Morita equivalence yields an isomorphism $$W^\pm(Q_m\text{-}\bold Q)\simeq\bigoplus_T W^+({\rm End}_{Q_m}T),$$ where T runs over all isomorphism classes of simple Q_m-modules, finite-dimensional over $\bold Q$ and admitting a non-singular $\pm$-Hermitian form θ; the involution on ${\rm End}_{Q_m}T$ is defined by $\theta(\overline f(x),\cdot)= \theta(x,f(\cdot))$ for some choice of θ. (This isomorphism associates to the Witt class of each (N_i,ϕ_i) such that $N_i\simeq T$ the Witt class of $({\rm Hom}_{Q_m}(T,N_i),\phi^\theta_i)$, where ϕ^θ_i is defined by $\theta(\phi^\theta_i(\alpha,\beta)(x), \cdot)= \pm\phi_i(\beta(x),\alpha(\cdot))$.) By the Schur lemma, each $E\coloneq{\rm End}_{Q_m}T$ is a division ring.

In the case of knots T runs over $\bold Q[s]$-modules $\bold Q[s]/(p)$ where p is an irreducible polynomial such that $p(s)=\pm p(1-s)$, and E is again $\bold Q[s]/(p)$ with $\overline s=1-s$.

Algorithmic Witt classification of Hermitian forms over the finite-dimensional division $\bold Q$-algebra E is known. For each of a certain five classes of such algebras, a complete list of Witt invariants is given by some combination of: the dimension modulo 2, the signatures (i.e., at most $[E\colon\bold Q]$ integers), the discriminant, the Hasse--Witt invariant and the Lewis θ-invariant, which is only defined when all the other invariants vanish. In contrast to the other invariants, the θ-invariant is non-local, i.e., does not factor through tensoring by $\bold Q_p$'s. It is only needed for the class of quaternionic algebras with the ``non-standard'' involution $\overline i=-i$, $\overline j=j$. Being noncommutative, they do not occur as $\bold Q[s]/(p)$; nor do commutative algebras with trivial involution, which are the only class requiring the Hasse--Witt invariant. (To be precise, $\bold Q[s]/(s-\frac12)$ does occur among them, but is not induced from any $\bold Z[s]$-module.)

In contrast, the author shows that for each $m>1$ and each sign ($+$ or $-$), every finite-dimensional division $\bold Q$-algebra with involution occurs as ${\rm End}_{Q_m}(T)$ for infinitely many T's as above, induced from P_m-modules. Starting from $\bold Q(\sqrt{-7})$ with the trivial involution, he exhibits explicit 8×8 Seifert matrices of 2-component links which have order 8 in C^5_2 and C^7_2.

Reviewed by Sergey A. Melikhov

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Non-commutative characteristic polynomials and Cohn localization. (English summary)

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For a noncommutative ring A the generalised characteristic polynomial of an endomorphism α of a finitely generated projective A-module P is defined to be the Whitehead torsion $[1 - \alpha X] \in K_{1}(A[[X]])$. The purpose of this paper is to give an example of A,P and α as above for which the generalised characteristic polynomial does not determine (P,α) up to extensions. This is in contrast to the situation when A is commutative, where the positive result was proved in [G. Almkvist, J. Algebra **28** (1974), 375--388; MR0432738 (55 #5721)]. The noncommutative pathology is explained in terms of the failure of a certain Cohn localisation of $A[X]$ to embed in $A[[X]]$. The latter localisation is that defined in terms of inverting matrices, as described in, e.g., Chapter 7 of [P. M. Cohn, *Free rings and their relations*, Second edition, Academic Press, London, 1985; MR0800091 (87e:16006)].

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