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Algebraic L-theory and topological manifolds. (English)

Cambridge Tracts in Mathematics. 102. Cambridge: Cambridge University Press. ix, 358 p. (1992). [ISBN 0-521-42024-5/hbk]

According to *C. T. C. Wall*, the algebraic L-theory means the algebraic K-theory of quadratic forms over a suitable ring (the quadratic forms considered up to stable isomorphism). In applications to topology, the ring is the group ring $\mathbb{Z}[\pi]$ of the fundamental group π of a manifold in question. The algebraic L-theory relates the topology of manifolds to their homotopy types.

The book under review provides a self-contained account of this relationship in dimensions ≥ 5, which was established by the Browder-Novikov-Sullivan-Wall surgery theory for compact differentiable and PL-manifolds, and extended to topological manifolds by Kirby and Siebenmann. More specifically, the book deals with the 'manifold structure existence problem' which is to decide if a finite Poincaré space is homotopy equivalent to a compact manifold, and the 'manifold structure uniqueness problem' which is to decide if a homotopy equivalence of compact manifolds is homotopic to a homeomorphism (or at least, h-cobordant to a homeomorphism).

The algebraic surgery theory of the author [see, e.g., The algebraic theory of surgery. I: Foundations, Proc. Lond. Math. Soc., III. Ser. 40, 87-192 (1980; Zbl 0471.57010), The algebraic theory of surgery. II: Applications to topology, ibid. 193-283 (1980; Zbl 0471.57011), Lower K- and L-theory, Lond. Math. Soc. Lect. Note Ser. 178 (1992)] is extended in the book to a combinatorial treatment of the manifold structures existence and uniqueness problems. The treatment provides an intrinsic characterization of the manifold structures in a homotopy type in terms of algebraic transversality properties on the chain level (for finite Poincaré spaces). The passage from the topology of compact manifolds to the homotopy theory of finite Poincaré spaces is the assembly (a passage from a local input to a global output) of basic interest in the book. In particular, the rigidity of aspherical manifolds with fundamental group π is shown to be equivalent to the fact that for the classifying space $B\pi$, the algebraic L-theory assembly map is an isomorphism.

The algebraic L-theory assembly map is constructed as a forgetful map between two algebraic Poincaré bordism theories, in which the underlying chain complexes are the same (but differ in the duality conditions). The assembly of a local algebraic Poincaré complex is a global algebraic Poincaré complex. Surgery theory identifies the fibre of the algebraic L-theory assembly map with the fibre of the assembly map from compact manifolds to finite Poincaré spaces in dimensions ≥ 5. This allows to create the homotopy types of compact manifolds out of the homotopy types of finite Poincaré spaces and some extra chain level Poincaré duality, and consequently, to obtain the central result of the book: the identification of manifold structure in the homotopy type of a Poincaré duality space with a local quadratic structure in the chain homotopy type of the universal cover. Also, the algebraic L-theory assembly map is used to give a purely algebraic formulation of the Novikov conjectures on the homotopy invariance of

the higher signatures. Moreover, there are applications to the topology of singular and stratified spaces, as well as to group actions on manifolds.
In short, the book presents the definitive account of the algebraic applications to the surgery classification of topological manifolds by using the abstract theory of quadratic forms on chain complexes developed by the author for understanding the connection between quadratic forms and manifolds. The book is published in the series "Cambridge Tracts in Mathematics", and according to the General Editors, Tracts are expected to be rigorous, definite, and of lasting value to mathematicians working in the relevant disciplines. There is no doubt that the present book fulfills all these expectations. The book is designed as an introduction to the subject, and it is divided into two parts, called 'Algebra' and 'Topology'. No previous knowledge of surgery theory is assumed, and every algebraic concept is justified by its occurrence in topology. In particular, the reader need not be familiar with other books on surgery theory such as those by *W. Browder* [Surgery on simply connected manifolds (1972; Zbl 0239.57016)], *C. T. C. Wall* [Surgery on compact manifolds, Academic Press (1970; Zbl 0219.57024)], and the author [Exact sequences in the algebraic theory of surgery, Mathematical Notes 26 (1981; Zbl 0471.57012)].

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Keywords : group ring of the fundamental group of a manifold; manifold structure existence problem; manifold structure uniqueness problem; algebraic L- theory; algebraic K-theory of quadratic forms; quadratic forms considered up to stable isomorphism; finite Poincaré space; homotopy equivalent to a compact manifold; h-cobordant to a homeomorphism; algebraic surgery theory; manifold structures in a homotopy type; algebraic transversality properties; rigidity of aspherical manifolds; algebraic L-theory assembly map; algebraic Poincaré bordism theories; Novikov conjectures; topology of singular and stratified spaces; group actions on manifolds; quadratic forms on chain complexes

Classification :

∗57-02 Research monographs (manifolds)
57R67 Surgery obstructions, Wall groups
57S17 Finite transformation groups
57P10 Poincare duality spaces
55N15 K-theory (algebraic topology)
18F25 Algebraic K-theory, etc.
19J25 Surgery obstructions (K-theory)
57N80 Stratifications
55M35 Finite groups of transformations

Cited in ...