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★**Algebraic and geometric surgery.**

Oxford Mathematical Monographs.
Oxford Science Publications.
The Clarendon Press, *Oxford University Press*, *Oxford*, 2002. *xii*+373 *pp. ISBN* 0-19-850924-3

Surgery theory, loosely speaking, refers to a variety of algebraic and geometric techniques used to classify manifolds, typically of dimensions 4 or greater. The theory encompasses topological classification within a homotopy type, existence and uniqueness of smooth or PL structures, and many other topics such as embeddings or automorphisms of manifolds. The term surgery itself refers to the process of cutting out a piece of a manifold (typically of the form $S^k \times D^{n-k}$) and replacing it with another (typically $D^{k+1} \times S^{n-k-1}$). This innocent-seeming operation becomes very powerful when combined with other tools such as bundle theory, handlebody theory (particularly the S-cobordism theorem) and the algebra of quadratic forms. The full complexity of the theory is seen when one is dealing with non-simply-connected spaces, and the calculation of the set $\mathcal{S}(X)$ of smooth n-manifolds homotopy equivalent to a space X is summarized in the "surgery exact sequence":

$$\cdots \to [\Sigma X, G/O] \to L_{n+1}(\mathbf{Z}[\pi_1 X]) \to \mathcal{S}(X) \to [X, G/O] \to L_n(\mathbf{Z}[\pi_1 X])$$

Of course, such an exact sequence per se is never enough to do real calculations; one must calculate and understand all of the terms in the sequence and the maps between them.

Ranicki's book provides an introduction to these ideas, pitched at a reader who knows the basics of algebraic topology and manifold theory. As such, it provides much more geometric background than the classic books on the subject [W. Browder, *Surgery on simply-connected manifolds*, Springer, New York, 1972; MR0358813 (50 #11272); C. T. C. Wall, *Surgery on compact manifolds*, Academic Press, London, 1970; MR0431216 (55 #4217)], from which at least two generations of practitioners have learned the subject. Wall's book, in particular, is a difficult (but rewarding) read that gives an impression of a subject just reaching its full power. (The second edition [*Surgery on compact manifolds*, Second edition, Amer. Math. Soc., Providence, RI, 1999; MR1687388 (2000a:57089)] has some useful commentary and updates by Ranicki.) The surgery sequence appears about halfway through Wall's book, and is followed by several dense chapters giving

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classification of manifolds in various homotopy types (tori, projective spaces, lens spaces) as well as applications to embeddings of manifolds. By contrast, the surgery sequence is the culmination of the book under review, which takes its time developing the geometric background and the algebra necessary to define the surgery groups, $L_n(\mathbf{Z}[\pi_1 X])$.

The treatment of the surgery groups, especially for n odd, differs from the original treatment in Wall's book. When n is even, the surgery groups are defined as equivalence classes of quadratic forms, which in turn are defined in terms of $\mathbf{Z}[\pi_1 X]$-valued intersection numbers. The detailed discussion of these intersection numbers (and the more subtle self-intersections) are an attractive feature of Ranicki's book. When n is odd, the definition of L_n is given in terms of pairs of Lagrangians in a standard $\mathbf{Z}[\pi_1 X]$-valued quadratic form. This definition, apparently suggested by S. P. Novikov [Math. USSR-Izv. **4** (1970), 257–292; ibid. **4** (1970), 479–505; translated from Izv. Akad. Nauk SSSR Ser. Mat. **34** (1970), 253–288; ibid. **34** (1970), 475–500.; MR0292913 (45 #1994)], is equivalent to that given in Wall's book but is somewhat easier to digest. Except for $\pi_1 = \{1\}$, there are no actual calculations of surgery groups in the book. However, some basic tools such as localization and the author's theory of "algebraic surgery" are briefly discussed, and references to the literature are given.

All in all, Ranicki's book is a readable introduction to this powerful theory that will be useful to a student or beginning user. One thing that such a reader should know, however, is that many of the proofs of the background results are sketched rather than being given in detail. The author's own advice (from the preface) is sound: start with the classic paper by M. A. Kervaire and J. W. Milnor [Ann. of Math. (2) **77** (1963), 504–537; MR0148075 (26 #5584)] and then move on to the more sophisticated versions. This book provides a very good next step. *Daniel Ruberman* (1-BRND)