Ranicki, Andrew: 57010
The algebraic theory of surgery. I: Foundations.
Proc. Lond. Math. Soc., III. Ser. 40, 87-192 (1980).

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The algebraic theory of surgery. II: Applications to topology.
Proc. Lond. Math. Soc., III. Ser. 40, 193-283 (1980).

[In this review the reviewer has included the author's recent book "Exact sequences in the algebraic theory of surgery". For a detailed review of this monograph see the following review.]

In these papers the author develops a surgery theory for chain complexes satisfying a duality condition. The original surgery theory involves modification of manifolds, and there is an obstruction involving quadratic forms. The chain complex theory is a natural half-way step between the geometry and the algebra. With the publication of "Exact sequences" the author has carried the theory far beyond the status of a useful intermediate step. The reviewer feels that it is now the best setting for the study of surgery as a theory. This is not to suggest a victory of algebra over geometry. Almost the opposite, since these chain complexes behave very much like spaces (with some new operations, like desuspending mapping cones). In fact intuitions from both points of view are useful in this context. The theory offers simplicity, completeness, and new applications. There is a lot of foundational material, but it is certainly less involved than the transversality, homotopy, bundle theory, immersion theory, etc. involved in the manifold approach. Of course these things are necessary for geometric applications, but it is an advantage not to have to deal with them all when investigating the theory. The foundations are summarized in the first chapter of "Exact sequences", with references to "Foundations". The treatment is comprehensive and detailed. In "Exact sequences" some of the major theorems of surgery are proved from this point of view, in versions the author describes with considerable justification as definitive. There are many notes (and Part II) relating the material to the geometry, and the bibliographical references are extensive and up to date. The arguments are also given in great detail. Although a burden for the publishers, this is very useful to those trying to extend the arguments or adapt them to new settings. Others can skip them. New applications probably will occur on three levels of abstraction. The lowest level exploits the direct access the theory gives to individual obstructions. A number of people has used it to check examples during development of general theorems. It is used in Part II, Section 8, to prove a very general product formula, and in Section 9 to investigate the dependence of manifold surgery obstructions on bundle data. The next level concerns the obstruction groups. Most of the known systematic computational techniques are developed in "Exact sequences". These include previously geometric material (polynomial extensions, low codimensional submanifolds) as well as algebraic (localization, Mayer-Vietoris sequences). As we move toward a deeper understanding of the groups it seems likely that one of the K-theory variants will prove more regular than the others. If it is the projective theory L^p, as seems likely, then the chain complex point of view will be nearly essential for further progress. There are geometric interpretations of these groups (noncompact surgery, surgery XS') but these are too awkward to use directly. The highest level of obstruction concerns the theory as a whole. When done on the classifying space level the L_* groups appear as homotopy groups of an infinite loop space. This rich structure has been only lightly exploited. Ranicki introduces the symmetric groups L^*, which are a ring over which L_* is a module. Presumably these are homotopy groups of an E^∞ ring spectrum [P. J. May, E_∞ ring spaces and E_∞ ring spectra (1977; Zbl. 345.55007)] over which the L spaces are modules. The bare existence of some of this structure has been used by L. Taylor and B. Williams [Algebr. topology, Proc. Conf., Waterloo/Canada 1978, Lect. Notes Math. 741, 170-198 (1979)] to derive formulae for surgery obstructions. There is undoubtably a great deal more to be done with these structures. For this the chain complex theory will be essential, since there is no satisfactory geometric discription of L^*. Readers are advised to start with Section 1 of "Exact sequences", and work toward particular goals from there. There are quite a few misprints. This is a considerable disadvantage since proofs tend to be formulae given explicitly, rather than reasoning from which formulae can be derived. Comparison of versions given in "Exact sequences" and Part I often resolves errors, but there are places where there are errors in both versions. Also changes in notation have confused matters. The author has a partial list of misprints.

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Exact sequences in the algebraic theory of surgery.
Mathematical Notes, 26. Princeton, New Jersey: Princeton University Press; University of Tokyo Press. XVII, 863 p. $ 21.50 (1981).
This book is Part III of a series of research papers of the author about "The

algebraic theory of surgery". In Part I: "Foundations" and Part II: "Applications to topology" [Proc. Lond. Math. Soc., III. Ser. 40, 87-192, 193-283 (1980; see the preceding review)] the cobordism theory of algebraic Poincaré complexes with a symmetric (resp. quadratic) structure was used to define for any ring A with involution a sequence of abelian groups $L^n(A)$ (resp. $L_n(A)$), $n \in \mathbb{Z}$, and to study their applications to the geometric theory of surgery on compact manifolds. The present Part III establishes various exact sequences for the L-groups, which show, that the groups $L^n(A)$ (resp. $L_n(A)$) are related to the symmetric (resp. quadratic) Witt group $L^0(A)$ (resp. $L_0(A)$) in a similar way as the higher algebraic K-groups are related to the projective class group $K_0(A)$. - §1 recapitulates the main definitions and results from Part I and Part II. In § 2 relative symmetric (resp. quadratic) L-groups $L^n(f)$ (resp. $L_n(f)$) are defined for a morphism $f: A \to B$ of rings with involution, and it is shown, that these groups fit into a long exact Part sequence $\ldots \to L^n(A) \to L^n(B) \to L^n(f) \to L^{n-1}(A) \to \ldots$ (similar for the quadratic L-groups). In § 3 the special case, where $f: A \to S^{-1}A$ is a localization map inverting a multiplicative involution invariant subset S of A of non-zero divisors, is considered and the groups $L^n(f)$ (resp. $L_n(f)$) are identified with the cobordism groups $L^n(A,S)$ (resp. $L_n(A,S)$) of symmetric (resp. quadratic) Poincaré complexes over A, which become acyclic over $S^{-1}A$. For $n \leq 1$ these L-groups are then interpreted as Witt groups of non-singular $S^{-1}A/A$-valued linking forms and linking formations on S-torsion A-modules of homological dimension 1. Thus the author obtains a generalization of localization exact sequences proved previously by M. Karoubi [Ann. Sci. Éc. Norm. Supér., IV. Sér. 7, 359-403 (1975; Zbl. 325.18011) and ibid. 8, 99-155 (1975; Zbl. 345.18007)], W. Pardon [Algebr. K-theory, Proc. Conf. Evanston 1976, Lect. Notes Math. 551, 336-379 (1976; Zbl. 362.15016)] and G. Carlsson and R. J. Milgram [J. Pure Appl. Algebra 18, 233-252 (1980; Zbl. 443.10019)]. The localization sequence is also used to prove that excision holds for $L^n(A,S)$ and $L^n(\hat{A},\hat{S})$ (resp. $L_n(A,S)$ and $L_n(\hat{A},\hat{S})$), where $\hat{A} = \varprojlim_{s \in S} A/sA$ is the S-adic completion of A. Thus the exact Mayer-Vietoris sequence $\ldots \to L^n(A) \to L^n(S^{-1}A) \oplus L^n(\hat{A}) \to L^n(\hat{S}^{-1}\hat{A}) \to L^{n-1}(A) \to \ldots$ (similar for the quadratic L-groups) attached to the arithmetic square

$$\begin{array}{ccc} A & \to & S^{-1}A \\ \downarrow & & \downarrow \\ \hat{A} & \to & \hat{S}^{-1}\hat{A} \end{array}$$

is obtained. Again, in special cases this sequence had been proved by C. T. C. Wall [Invent. Math. 23, 261-288 (1974; Zbl. 278.16018)] and Karoubi (loc. cit.). A similar sequence in unitary K-theory has been obtained by A. Bak [K-theory of forms (1981; Zbl. 465.10013)]. In § 4 the author applies his techniques to the classical situation of a Dedekind domain, discusses the failure of reduction for L^0 and of devissage for L_0 and computes the integral and rational L-groups. § 5 is devoted to the "Fundamental theorem of algebraic L-theory": If A is a ring with involution $^-$, α a ring automorphism of A, such that $\overline{\alpha(a)} = \alpha^{-1}(\bar{a})$ $(a \in A)$ and x an indeterminate with $\bar{x} = x$, $ax = x\alpha(a)$, the L-groups of the α-twisted polynomial extensions $A_\alpha[x]$, $A_\alpha[x,x^{-1}]$ are related by exact sequences of the type $0 \to L^n(A) \to L^n(A_\alpha[x]) \oplus L^n(A_\alpha[x^{-1}]) \to L^n(A_\alpha[x,x^{-1}] \to L^n(A^\alpha) \to 0$ (similar for the quadratic L-groups), where A^α denotes the ring A with involution $a \to \alpha(\bar{a})$. In § 6 exact Mayer-Vietoris sequences are derived for quadratic L-groups, which are attached to a cartesian square

$$\begin{array}{ccc} A & \to & B' \\ \downarrow & & \downarrow \\ B & \to & A' \end{array}$$

of rings with involution, where $B' \to A'$ or $B \to A'$ is surjective. In general, there are no such sequences for the symmetric L-groups. Finally, § 7 is viewed as a preliminary step towards codimension q surgery theory with further applications to be expected in forthcoming papers of the author. This book, which is principally a research monograph, is very useful for all people working in surgery theory, since it presents in a clearly written way all possible exact sequences, that are needed in attacking surgery problems, and they certainly will appreciate the great effort done by the author to lay firm ground in this part of mathematics. Hopefully, this book will also be recognized by mathematicians working in more classical quadratic form theory, since the tools presented here seem to be of great use for the solution of problems about quadratic forms over arbitrary rings. A first step in this direction has recently been done by Pardon [The map of the Witt group of a regular local ring to the Witt group of its field of fractions (preprint)].

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